

# Path Tracking and Obstacle Avoidance for a Miniature Robot

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## Abstract

Path tracking and obstacle avoidance are two very important behaviours that must be considered in the process of developing Autonomous Ground Vehicles, AGV:S. An AGV is a vehicle with the ability to operate all by itself at ground level, that is an unmanned vehicle under the control of a computer. A lot of progress has been done in the field of developing AGV:s in the last decade, and successful applications has been made in both mining and agriculture. Now the research is starting to concentrate on developing an AGE that can operate in forest-like environments. It is in this kind of challenging environments that path tracking and especially obstacle avoidance becomes very important issues.

When in path tracking mode, the vehicle is travelling along a memorized path under the complete control of a computer. The path has then been memorized by letting a human operator manually remote the vehicle along a predefined path. If the AGV during the process of path tracking detects any new obstacles on the path, the obstacle avoidance routine should find a new route around the obstacle and then return on the path.

In this project I will implement and evaluate algorithms for path tracking and obstacle avoidance on a miniature robot. The work is related to similar problems in the IFOR-project [4]. The robot I will be working with is the Khepera model, a two-wheeled differential steered small robot that uses eight proximity sensors and a camera module for detection of obstacles. I will study and evaluate the performance of existing well known path tracking algorithms, like follow-the-carrot and pure-pursuit, as well as the more recently developed vector-pursuit tracking method.

As for the obstacle avoidance behaviour, I noticed that a simple local algorithm would be suitable in this project, because of lacking precision in the Kheperas ability to detect obstacles. The Matlab environment is used for all implementation and testing, as well as the displaying of tracking routes and evaluations of the various methods.



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# 1 Introduction

Developing an unmanned vehicle with the ability to autonomously transport timber from the area of felling to the main roads for further transportation to the factories is not an easy task. Several important factors must be considered in the planning phase of such a project. Among the most important ones are of course the safety issues involved – a vehicle that large moving autonomously in a forest environment has to have very accurate sensors for detection of humans as well as other obstacles in the vicinity of the vehicle. In addition to this some good behaviours must be implemented in the on board computer for dealing with situations involving obstacles of any kind. Another important part of the project is the path tracking behaviour of the vehicle. This behaviour is made up of two parts. The first part is *the recording phase*, for which a human operator manually remotes the vehicle along a predefined path and a computer is recording all of the route travelled. Naturally the second phase is the *path tracking mode*, and in this mode the computer takes full control of the vehicle, following the recorded route as close as possible considering any biases that could accumulate position and heading errors along the way. During this second phase any obstacles detected in the direction of travel would either cause the vehicle to stop and return control to the driver, or in case of static obstacles like trees or stumps a detour around and then returning to the original track.

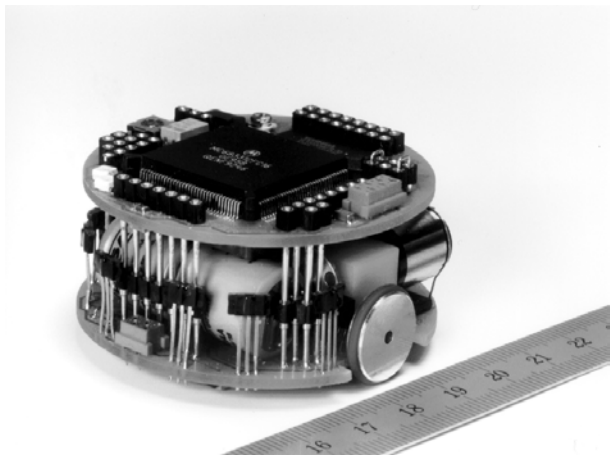
This project is related to the problems above. I will implement, test and evaluate a number of the existing path tracking algorithms available today, trying to figure out which of these would be most appropriate to integrate in a forest machine like the one mentioned previously. Although this tests will only be performed on a miniature robot called Khepera, the results gained can help ease things for the original IFOR-project.

I have divided the report in two main sections; the first part consists of all the theory related to the project, and in the second part I present and display the results gained from testing the methods in different situations, and evaluate them accordingly. In the first section I will initially describe the robot used for all experiments, then some important theory behind the localization of the robot in an unknown environment, followed by a brief explanation of the path tracking algorithms that has been implemented and tested on this robot. The evaluation part consists of display and analysis of four different types of paths. I have compared the methods to each other on basis of mean and maximal deviation from the path, depending on variables like speed and look-ahead distances.

## 2 The Khepera robot

### 2.1 Overview

Khepera is a miniature robot especially well suited for laboratory experiments in a school environment [8]. It was originally developed by the LAMI team of the EPF in Lausanne Switzerland, and it's commercially available at the K-Team S.A. Kheperas small size, only 5.5 cm in diameter, gives it a lot of advantages in many different areas. One advantage is the possibility to have the robot on a desk close to the computer when performing experiments and tests of different algorithms. A miniature robot of this kind further allows easy manipulation and reduces the effort needed for transportation to other locations. One also has the possibility to increase functions by adding extension turrets on top of the base unit. Below you can see a picture of this base module.

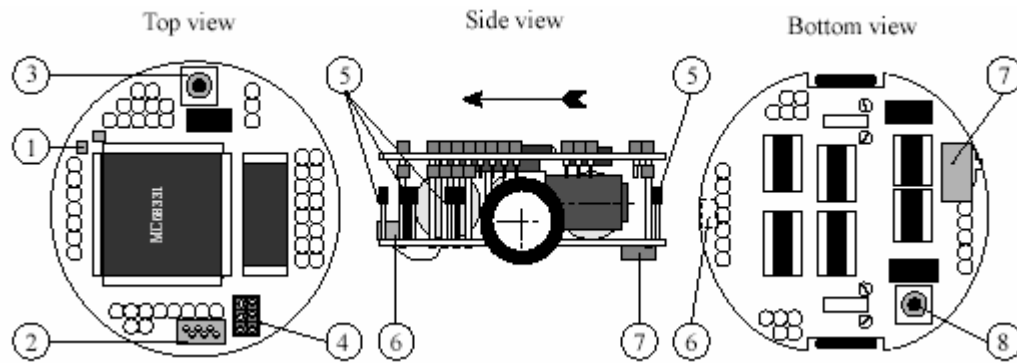


**Fig 1.** Khepera base module

The Khepera robot is only equipped with two wheels that does both the steering and propulsion. This kind of drive mechanism is called differential drive, and is often used on small miniature robots of this kind. Each of the two wheels is controlled by a DC motor through a 25:1 reduction gear box. An incremental magnetic encoder is mounted on each motor and returns 24 pulses per revolution of the motor, which allows a resolution of 600 pulses per revolution of the wheel. This corresponds to 12 pulses per millimetre of path of the robot or 0,08 millimetre per pulse, considering the wheel diameter. The maximum speed according to the specifications is 60 cm/s, and the minimum is 2 cm/s.

The standard processor type is a Motorola 68331 with a frequency of 38 MHz and a RAM consisting of 256 Kb. Its is powered either by an adapter through an S serial line connector or by rechargeable batteries. An overview of the Khepera robot from different angles is shown below:





**Fig 2.** Overview of some parts on the Khepera robot

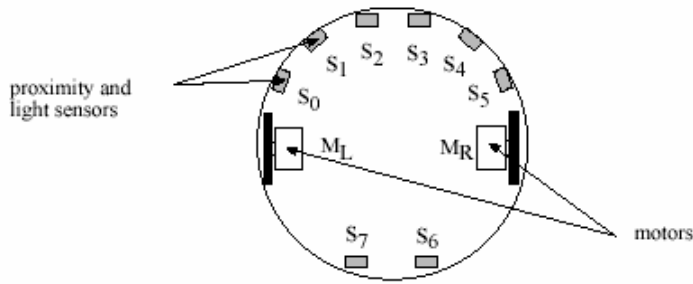
*Numbered parts:*

1. LEDs
2. Serial line(s) connector
3. Reset button
4. Jumpers for the running mode selection
5. Infra-Red proximity sensors
6. ON-OFF battery switch
7. Second reset button (same function as 3)

## **2.2 Sensors**

The Khepera is also equipped with eight proximity sensors placed around the base module according to fig 3. These sensors consist of an integrated IR emitter and receiver which allows two types of measurements:

- Ambient light measurement. This measurement is performed using only the receiver. The emitter is totally inactive. The sampler rate is 20 ms. Activation and reading of a single sensor takes 2.5 ms to complete.
- Measurement of light reflected from obstacles. This type of measurement is done by using both the emitter and the receiver. The difference between the two values is returned. The sample rate, and the activation and reading time are the same as for the ambient light measurement.

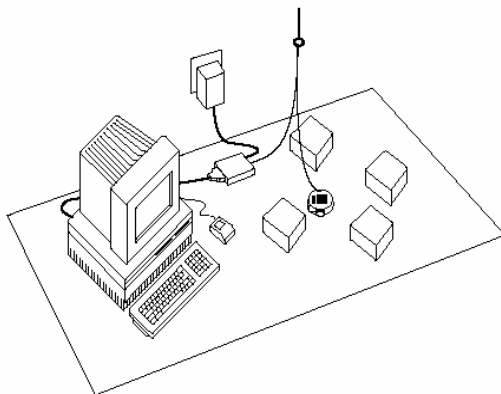


**Fig 3.** Position and orientation of sensors on the base module

The values returned from these proximity measurements depends on factors like colour, shape and intensity of the obstacle detected, as well as the distance to the light source. I noticed that white objects was preferable - the more darker an object the worse accuracy of measurement. At optimal conditions the proximity sensors can detect obstacles at a maximum distance of 50 mm, although in my experiments this distance surely wasn't greater than 30 mm. The values returned from proximity readings varies from 0 to 1024 - 0 indicating that no obstacle was detected, a larger value points to the opposite. The high frequency of noise in the readings is a disadvantage, making the measurements less reliable and the experiments more unstable. Noise results in readings indicating the false presence of obstacles, and is caused by light reflections.

### 2.3 Connections and communication

Only two connections is needed, one is used for recharging of batteries and the other for connection between robot and computer. Communication between robot and host computer is done through a RS232 line, which is converted by the interface module to a S serial line available on the robot. The S serial line connects the robot with the interface module, and is also responsible for the Kheperas power supply. Another connection is made between interface module and host computer by the RS232 line, and last but not least the interface module is connected to the power supply. This yields the communication configuration of the Khepera, which is also shown in figure 4.



**Fig 4.** Robot – computer Communication configuration

Robot and host computer communicates by sending and receiving ASCII messages. The host computer initiates communication by sending a command to the Khepera followed by a carriage return or a line feed. Then a response is sent by the robot to the host to confirm message was received and executed. This procedure corresponds to a master – slave protocol, with the host computer taking on the role as master and the robot gets to be the slave. All commands consists of a single capital letter, sometimes followed by numerical or literal parameters separated by a comma and terminated by a linefeed. Responses comes in the form of the command letter in lower case, followed by potential numerical and literal letters depending on the command sent.

### 3 Dead Reckoning

#### 3.1 Basics

*Dead reckoning* is a method used for determining the current position of a vehicle, by advancing some previous position through known course and velocity information over a given time period [1]. Or more simply put, a process of estimating the position of a vehicle based solely on speed, direction and time elapsed since the last known position. Dead reckoning was probably derived from the term *deduced reckoning* by sailors sometimes in the seventeenth century. Theory has it someone abbreviated this to ded reckoning in the ships log, and later on another unknown person must have thought it was a misspelling for *dead*. There are more theories behind the origin of the term, but this is the most commonly used today.

Most of the robotic ground vehicles today rely on dead reckoning as the framework of their navigational systems. One simple way to implement dead reckoning is called *odometry*. Odometry is a method to provide information about vehicle displacement based on the rotation of its motors or wheels. The rotation is measured by wheel or shaft encoders. These encoders counts the number of rotations of the wheel axles, providing data that combined with the forward kinematics equations can be translated to information regarding the change in position and heading of the vehicle. Figure 5 shows a picture of the normal configuration and placement of encoders and motors on a differential drive ground robot vehicle, like the Khepera.

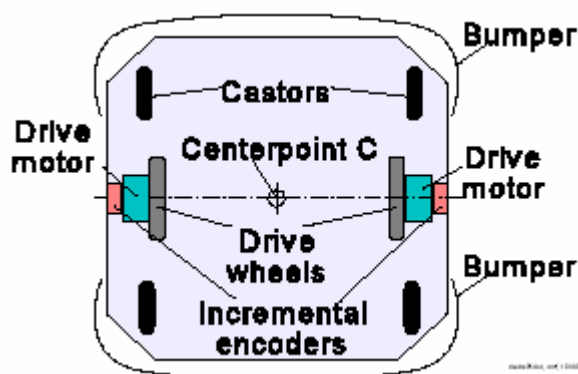
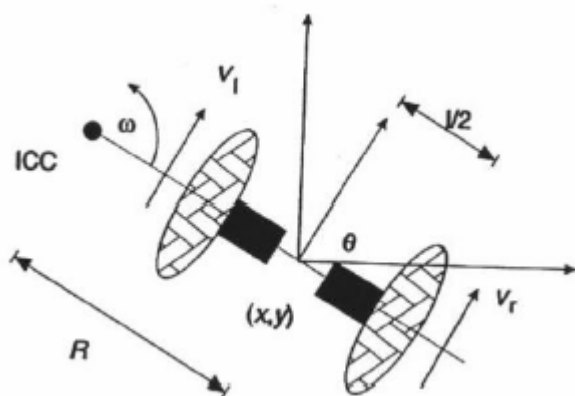


Fig 5. Differential drive mobile robot

What then is *forward kinematics*? Kinematics is the study of motion, but without having to be concerned with the forces that affect the motion. The problem of determining what positions that is reachable given only the velocities of the wheels is called forward kinematics.

### 3.2 Differential drive kinematics

The theory behind differential drive kinematics is pretty straightforward. Every wheeled mobile robot in a state of motion must always rotate about a point that lies somewhere on the common axis of the two wheels [6]. This point is often called the instantaneous centre of curvature (ICC), or the instantaneous centre of rotation (ICR). A differential drive robot, like the Khepera, changes the position of the ICC simply by varying the velocities of the two wheels. Two separate drive motors, one for each wheel, provide independent velocity control to the left and right wheels. It is this property that makes it possible for the robot to choose different trajectories and paths. A sketch describing the kinematics of a differential drive mobile robot can be seen in figure 6.



**Fig 6.** Differential drive kinematics

Each wheel follow a path that moves around the ICC at the same angular rate  $\omega$  [6], thus

$$\begin{aligned}\omega (R + l/2) &= v_r \\ \omega (R - l/2) &= v_l\end{aligned}$$

where  $l$  is the distance between the two wheels, the right wheel moves with velocity  $v_r$  and the left wheel moves with velocity  $v_l$ .  $R$  is the distance from the ICC to the midpoint between the wheels. All these control parameters are functions of time, which gives

$$R = l/2 * (v_l + v_r) / (v_r - v_l)$$

$$\omega = (v_r - v_l) / l$$

We have two special cases that comes from these equations. If  $v_l = v_r$ , then the radius  $R$  is infinite and the robot moves in a straight line. If  $v_l = -v_r$ , then the radius is zero and the

robot rotates in place. In all other cases the robot moves in a trajectory around the ICC at some angular rate  $\omega$ .

The forward kinematics equations can be derived easily now that we have established the basics. Our focus is on how the  $x$  and  $y$  coordinates and the orientation change with respect to time [2]. Let  $\theta$  be the angle of orientation, measured in radians, counter-clockwise from the  $x$ -axis. If we let  $m(t)$  and  $\theta(t)$  be functions of time representing speed and orientation for the robot, then the solution will be in the form:

$$dx/dt = m(t) \cos(\theta(t)) \quad (3.1)$$

$$dy/dt = m(t) \sin(\theta(t)) \quad (3.2)$$

The change of orientation with respect to time is the same as the angular rate  $\omega$ . Therefore

$$d\theta/dt = \omega = (v_r - v_l)/l \quad (3.3)$$

Integrating this equation yields a function for the robots orientation with respect to time. The robots initial orientation  $\theta(0)$  is also replaced by  $\theta_0$ :

$$\theta(t) = (v_r - v_l)t/l + \theta_0 \quad (3.4)$$

Since the velocity in functions (3.1) and (3.2) above simply equals the average speed for the two wheels, that is  $m(t)=(v_r+v_l)/2$ , integrating this in (3.1) and (3.2) gives:

$$dx/dt = [(v_r + v_l)/2] \cos(\theta(t)) \quad (3.5)$$

$$dy/dt = [(v_r + v_l)/2] \sin(\theta(t)) \quad (3.6)$$

The final step is to integrate equations (3.5) and (3.6), and taking the initial positions to be  $x(0) = x_0$ , and  $y(0) = y_0$  to get:

$$x(t) = x_0 + 1/2(v_r + v_l)/(v_r - v_l) [\sin((v_r - v_l)t/l + \theta_0) - \sin(\theta_0)] \quad (3.8)$$

$$y(t) = y_0 - 1/2(v_r + v_l)/(v_r - v_l) [\cos((v_r - v_l)t/l + \theta_0) - \cos(\theta_0)] \quad (3.9)$$

Noting that  $1/2(v_r + v_l)/(v_r - v_l) = R$ , the robots turn radius, and that  $(v_r - v_l)/l = \omega$ , equations (3.8) and (3.9) can be reduced to:

$$x(t) = x_0 + R[\sin(\omega t + \theta_0) - \sin(\theta_0)] \quad (4.0)$$

$$y(t) = y_0 - R[\cos(\omega t + \theta_0) - \cos(\theta_0)] \quad (4.1)$$

This is the theory that lies behind implementing dead reckoning on a wheeled mobile robot using differential steering. The only thing one has to do is to substitute the terms  $v_r$  and  $v_l$  with  $s_r$  and  $s_l$ , indicating the calculations of displacements rather than speeds, and as a result of this also drop the time value  $t$ . Here  $s_r$  and  $s_l$  are the distances traveled by the left and right wheels respectively. Finally when this has been done equations (3.8) and (3.9) becomes:

$$x(t) = x_0 + 1/2(s_r + s_l)/(s_r - s_l) [\sin((s_r - s_l)/l + \theta_0) - \sin(\theta_0)] \quad (4.2)$$

$$y(t) = y_0 - 1/2(s_r + s_l)/(s_r - s_l) [\cos((s_r - s_l)/l + \theta_0) - \cos(\theta_0)] \quad (4.3)$$

which are the forward kinematics equations used by differential drive vehicles when turning.

### 3.3 Pros and cons

What are the pros and cons of implementing odometry based dead reckoning as the navigational system on a wheeled mobile robot? Odometry is very inexpensive compared to other methods, like GPS or ground beacon systems. It doesn't require complex mathematical calculations, yet providing rather good short term accuracy. Another advantage is the high sampling rates, which further enhances the positional accuracy compared to methods with a slower sampling rate. But with so many other things in the world, there is also a bunch of disadvantages involved. One primary disadvantage with odometry is its tendency to accumulate errors over time. There are mainly two types of errors that can be accumulated in systems based on odometry: *systematic errors* and *non-systematic errors*. Systematic errors are caused by flaws in the mechanical design of the robot, that is internal model-specific errors most often created by unequal wheel diameters or uncertainty about what wheelbase is the most effective. Non-systematic errors on the other hand are caused by external factors and not by the kinematics properties of the vehicle. Wheel-slippage and rough floor surfaces containing irregularities are examples of this kind of errors. Systematic errors are worse than environment depended errors, because they accumulate constantly. Despite these drawbacks many researchers are convinced that dead reckoning still is an important part of robot navigation.

## 4 Path Tracking

Path tracking is the process concerned with how to determine speed and steering settings at each instant of time in order for the robot to follow a certain path. A path consists of a set of points representing the positional coordinates of a particular route. Often when implementing a path tracking algorithm, one also has to implement a path recording unit responsible for saving all the coordinates that constitutes the path. A human operator then has the possibility to manually steer the robot along some track while the path recording unit saves the information about the path. The path tracking algorithm also has to handle unplanned positional or orientation deviations from the path. Such deviations can be caused by odometric errors of some kind, or by new obstacles occurring on the path that must be avoided.

There are many different types of path tracking algorithms available today. I have chosen to implement and evaluate three of them on the Khepera robot; follow-the-carrot, pure pursuit and vector pursuit. The first two methods have been around for quite a while now, while vector pursuit or screw tracking as it's also called is relatively new on the scene. The big difference between these methods is that vector pursuit uses information about orientation at the look-ahead point, while the others don't.

## 4.1 Follow-the-carrot

This algorithm is based on a very simple idea. Obtain a goal-point, then aim the vehicle towards that point [10]. Figure 7 describes the basics behind this method.

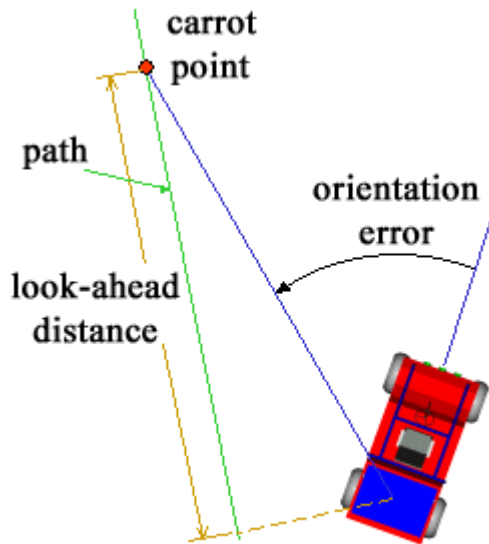


Fig 7. Follow-the-carrot

A line is drawn from the centre of the vehicle coordinate system perpendicular to the path. The carrot point, or goal point, is then defined to be the point on the path a look-ahead distance away from the intersection point of this line. The most important parameter is the orientation error, defined to be the angle between current vehicle heading and the line drawn from the centre of the vehicle coordinate system to the carrot point. A proportional control law then aims at minimizing the orientation error between the vehicle and the carrot point. An orientation error of zero means the vehicle is pointing exactly towards the carrot point. The magnitude of a turn  $\phi$  is decided by:

$$\phi = k_p * e_o$$

where  $k_p$  is the proportional gain and  $e_o$  is the orientation error. One also had the possibility of increasing the accuracy of the controller, perhaps adding integrative or derivative functions to it [11]. However, simple proportional controllers are still the most frequently used in this algorithm.

Although the follow-the-carrot approach is easy to understand and very simple to implement, it has a couple of major drawbacks. Firstly, the vehicle has a tendency to naturally cut corners. This happens because the vehicle immediately tries to turn towards each new carrot point. Another drawback with this path tracking technique is that the vehicle could oscillate about the path, particularly in the case of small look-ahead distances or at higher speeds. There are however modifications that can be made to the algorithm to increase its efficiency and accuracy. The selection of steering angle can be modified to be based on both the positional error displacement perpendicular to the path and the orientation error [11]. Yet the disadvantages of this method are generally too high, making it useless for implementation in

new projects that require good tracking ability. Still it is very suitable for educational purposes, or for comparison with other tracking algorithms.

## 4.2 Pure Pursuit

The concept of the pure pursuit approach is to calculate the curvature that will take the vehicle from its current position to a goal position [3]. The goal point is determined in the same manner as for the follow-the-carrot algorithm. A circle is then defined in such a way that it passes through both the goal point and the current vehicle position. Finally a control algorithm chooses a steering angle in relation to this circle. In fact, the robot vehicle changes its curvature by repeatedly fitting circular arcs of this kind, always pushing the goal point forward.

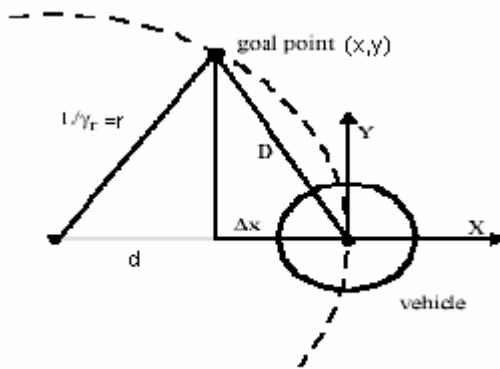


Fig 8. Pure Pursuit approach

It is important to note that the description of the pure pursuit algorithm in figure 8 is shown in vehicle coordinates. The vehicle coordinate system is defined where the y-axis is in the forward direction of the vehicle, the z-axis is down and the x-axis forms a right-handed coordinate system. Therefore all coordinates used must first be transformed to vehicle coordinates in order for the algorithm to work properly. Luckily it is pretty straight forward to convert coordinates located in one system into its representation in another system [9]. Let  $(x_r, y_r)$  be the current position of the robot, and  $(x_g, y_g)$  the goal point to be converted into vehicle coordinates. Then

$$\begin{aligned} x_{g_v} &= (x_g - x_r) \cos(\Phi) + (y_g - y_r) \sin(\Phi) \\ y_{g_v} &= -(x_g - x_r) \sin(\Phi) + (y_g - y_r) \cos(\Phi) \end{aligned}$$

where  $(x_{g_v}, y_{g_v})$  is the goal point in vehicle coordinates and  $\Phi$  is the current vehicle heading. In the figure above  $D$  is defined to be the distance between current vehicle position and the goal point.  $\Delta x$  is the x offset of the goal point from the origin, and  $1/\gamma_r$  is the radius of the circle that goes through the centre of the vehicle and the goal point. The required curvature of the vehicle is computed by:

$$\gamma_r = 2\Delta x / D^2$$

The derivation of this formula is based on just two simple equations [5]:



- 1)  $x^2 + y^2 = D^2$
- 2)  $x + d = r$

$(x,y)$  being the coordinates of the goal point in figure 8 above. The first equation is a result of applying Pythagoras' theorem on the smaller right triangle in the same figure, and the second equation comes from summing the line segments on the x-axis. The following derivation is pretty straightforward and should not be that difficult to understand:

$$\begin{aligned}
 d &= r - x \\
 (r - x)^2 + y^2 &= r^2 \\
 r^2 - 2rx + x^2 + y^2 &= r^2 \\
 2rx &= D^2 \\
 r &= D^2/2x \\
 \gamma_r &= 2x/D^2
 \end{aligned}$$

The last step comes from the mathematical fact that a circle has a constant curvature which is inversely proportional to its radius.

The name *pure pursuit* comes from our way of describing this method. We paint a picture in our minds of the vehicle chasing this goal point on the path a defined distance in front of it - it is always *in pursue* of the point. This can also be compared to the way humans drive their cars. We usually look at some distance in front of the car, and pursue that spot.

Assuming that the points that constitutes the path and the successive positions of the robot vehicle belongs to the same coordinate system, the pure pursuit algorithm can be described by these simple steps:

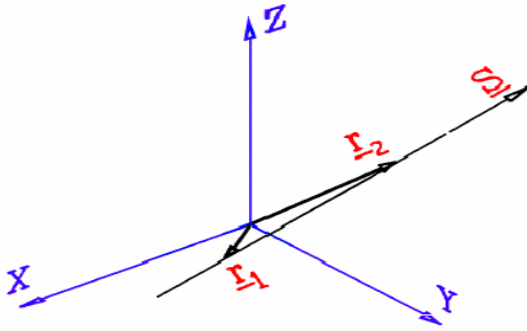
1. Obtain current position of the vehicle
2. Find the goal point:
  - 2.1. Calculate the point on the path closest to the vehicle  $(x_c, y_c)$
  - 2.2. Compute a certain look-ahead distance  $D$
  - 2.3. Obtain goal point by moving distance  $D$  up the path from point  $(x_c, y_c)$
3. Transform goal point to vehicle coordinates
4. Compute desired curvature of the vehicle  $\gamma = 2\Delta x/D^2$
5. Move vehicle towards goal point with the desired curvature
6. Obtain new position and go to point 2

The pure pursuit technique shows better results than the follow-the-carrot method described earlier. One improvement is less oscillations in case of large-scale positional and heading errors, another is improved path tracking accuracy at curves. Because of the advantages this method is more frequently used in real world applications than the follow-the-carrot algorithm. Other optimized methods may show better tracking results in some areas, but overall this is yet the best algorithm available today.

### 4.3 Vector Pursuit

Vector pursuit is a new path tracking method that uses the theory of screws first introduced by Sir Robert S. Ball in 1900. Screw theory can be used to represent the motion of any rigid body in relation to a given coordinate system, thus making it useful in path tracking applications. Any instantaneous motion can be described as a rotation about a line in space with an associated pitch. Screw control was developed in an attempt to not only have the vehicle arrive at the goal point, but also to arrive with the correct orientation and curvature. The methods described earlier, follow-the-carrot and pure pursuit, does not use the orientation at the look-ahead point. However vector pursuit uses both the location and the orientation of the look-ahead, giving it an advantage over the other algorithms as it ensures that the vehicle arrives at the current goal point with the proper steering angle.

A screw consists of a centreline and a pitch defined in a given coordinate system. The centreline can be defined by using Plücker line coordinates. A line can be represented using only two points represented by the vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . This line can also be defined as a unit vector  $\mathbf{S}$ , in the direction of the line and a moment vector  $\mathbf{S}_0$ , of the line about the origin. This representation can be seen in figure 9.



**Fig 9.** A line represented by two vectors

Based on the Plücker line representation we then have:

$$\mathbf{S} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

$$\mathbf{S}_0 = \mathbf{r}_1 \times \mathbf{S}$$

where the vectors  $(\mathbf{S}; \mathbf{S}_0)$  are the Plücker line coordinates of this line. Then by defining  $\mathbf{S} = [L, M, N]^T$  and  $\mathbf{S}_0 = [P, Q, R]^T$ , with  $\mathbf{r}_1 = [x_1, y_1, z_1]$  and  $\mathbf{r}_2 = [x_2, y_2, z_2]$  we have:

$$L = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

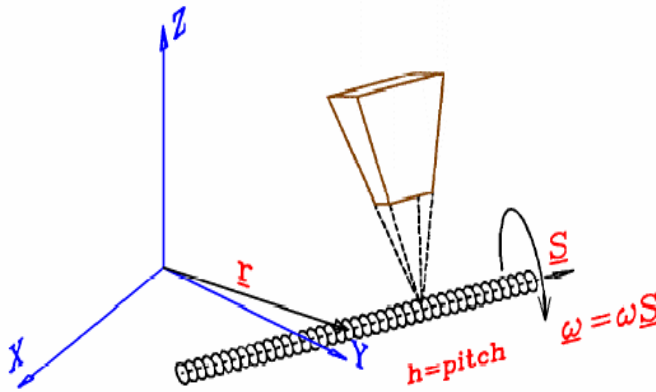
$$M = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$N = \frac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

and

$$\begin{aligned} P &= y_1 N - z_1 M \\ Q &= z_1 L - x_1 N \\ R &= x_1 M - y_1 L \end{aligned}$$

As explained earlier any instantaneous motion of a rigid body can be described as a rotation about a line in space with an associated pitch. Figure 10 shows such a body rotating with an angular velocity  $\omega$ , about a screw  $\underline{S}$  defined by its centreline ( $\underline{S}$ ;  $\underline{S}_0$ ). The pitch of the screw is also shown in the figure.



**Fig 10.** Instantaneous motion about a screw

The velocity of a rigid body depends on both the velocity due to the rotation and the translational velocity due to the pitch of the screw. This velocity is represented as:

$$\omega \underline{S} = (\omega \underline{S}; \omega \underline{S}_{oh}) \quad (4.3.1)$$

where

$$\underline{S}_{oh} = \underline{S}_0 + h \underline{S} = \underline{r} \times \underline{S} + h \underline{S}$$

and  $\underline{r}$  is any vector from the origin to the centreline of the screw.

The vector pursuit method developed by Wit (2001) calculates two instantaneous screws. The first screw accounts for the translation from the current vehicle position to the look-ahead point, while the second screw represents the rotation from the current vehicle orientation to the orientation at the look-ahead point. These two screws are then added together to form the desired instantaneous motion of the vehicle. This information is then used for calculating the desired turning radius that will take the vehicle from its current position to the goal point.

Pure translation of a rigid body is defined as the motion about a screw with an infinite pitch, and therefore (4.3.1) becomes  $v_{\underline{S}} = (0 ; v_{\underline{S}})$ . Pure rotation on the other hand is defined by the motion about a screw with a pitch equal to zero, and (4.3.1) reduces to  $\omega_{\underline{S}} = (\omega_{\underline{S}} ; \omega_{\underline{S}_o})$ . Then depending on whether the nonholonomic constraints of the vehicle are initially ignored or not, the screws with respect to translation and rotation are calculated and summed up to form the desired instantaneous motion of the vehicle.

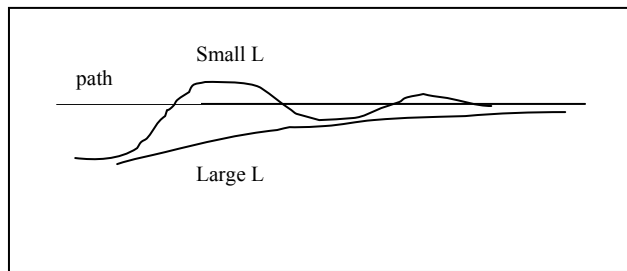
The results of the testing performed by Wit [3] both on real vehicles as well as in simulation shows comparable tracking results to the other methods described earlier. Vector pursuit showed particularly good skills in handling jogs appearing suddenly in the middle of the path. This means the method is doing rather good jumping from a small error in position and orientation to fairly large errors. Similar situations can occur in the real world when the robot vehicle detects sudden obstacles appearing on the path, forcing the vehicle to drive around the obstacle and then return to the path. It can also occur if there is noise in the position estimations, which can happen if the vehicle was using GPS techniques for localizing. In other test cases vector pursuit shows comparable results to the pure pursuit algorithm. This is not so surprising, because both methods calculates a desired radius of curvature and the centre point of rotation when tracking a curved path.

#### **4.4 The look-ahead distance**

Common to all these methods is that they use a look-ahead point, which is a point on the path a certain distance  $L$  away from the orthogonal projection of the vehicles position on the path. Path tracking techniques that uses this look-ahead point are called geometric algorithms. Changing the look-ahead distance can have a significant effect on the performance of the algorithm. Increasing the value  $L$  tends to reduce the number of oscillations, thus ensure a smooth tracking of the path. However this will also cause the vehicle to cut corners, severely reducing the tracking precision of curvy paths. The effects of changing this parameter will in the end be a question of in what context you want the tracker to provide good results. There are two problems that need to be considered:

- I. Regaining a path
- II. Maintaining the path

The first problem occurs when the vehicle is way off the path, thus having large positional and orientation errors, and is trying to return to this path. Under this circumstances it is pretty obvious what the effects will be of changing the look-ahead distance. A larger value causes the vehicle to converge more smoothly and with less oscillations. On the other side, once back on the path a large value will lead to worse tracking, especially in the case of paths containing very sharp curves. So a difficult trade-off must often be made when choosing the look-ahead distance for a particular path tracking algorithm. Do I expect the vehicle to encounter obstacles while in tracking mode, causing large positional errors? Or is it more likely the path will be obstacle free but rather curvy? These are questions that must be considered before implementing any systems of this kind.



**Fig 11.** The effects of having a small look-ahead distance contra a large for problem 1

Another factor that must be considered when choosing look-ahead distance  $L$  is the vehicle speed. An increase in vehicle speed would also require the distance  $L$  to be increased. The reason for this is that higher speeds requires the vehicle to start turning at an earlier stage. Generally the vehicle always starts turning before the curve at every look-ahead distance larger than zero. This is a positive quality that compensates for the time needed in the execution phase of the turn. Good tracking ability at high speeds is of great importance, and therefore a high priority for real world robot vehicle applications today. As everyone knows, time and money goes hand in hand. Tracking a path in five minutes is better than to do it in ten, and doing so while not losing accuracy is even better.

## 5 Implementation issues

### 5.1 Dead-reckoning unit

The first thing I implemented was the dead-reckoning module [3.1]. All position estimation for the Khepera robot is based on simple odometry, and the differential drive steering and propulsion system made the coding of this unit pretty straightforward. The basics and theory behind implementing dead-reckoning on this type of robot is thoroughly described by G.W Lucas [2]. This is the ground work of the whole project, since every other unit uses it for position estimation and updating.

There are basically three separate potential types of motion that must be updated at all time by this unit. These are rotation, turning and straight forward moving. In case of rotation, the vehicle is standing on the same spot rotating around some point by having one of the wheels spinning forward at some speed  $v$  while the other wheel is spinning backwards at the same speed  $v$ . This means only the vehicles heading or angular direction must be updated during this phase. On the same basis, straight forward moving means only position needs to be updated, not the heading angle.

The tricky part is the turning phase, which means both position and heading is changing at the same time. This calls for some mathematical calculations and derivations before coming up with the formulas required for the calculations. I described this derivation in chapter 3.2 on page 12. What's important to understand is that at fixed wheel velocities, the vehicle always

follows a circular path, thus deciding the turn radius of this circular trajectory becomes an important part of the equations behind the turning phase.

## 5.2 Remote unit

After finishing the dead reckoning unit the logical conclusion was to start implementing the remote unit. The major purpose of this one is to give the user the possibility to remote the Khepera robot by means of the keyboard. Thus the human operator can feel free to create his or her own path to be tracked by the path tracking units. During the execution phase of this unit, all coordinates constituting the path will be recorded in a matrix, that means both position and heading coordinates at every new position reached by the robot.

The function specification of the remote unit looks as follows:

```
path = remote(port, startpos)
```

where *path* is the variable used to store all coordinates, *port* is the communication inport number and *startpos* is the initial starting position of the robot in world coordinates. *startpos* is a vector in the form  $[x, y, \theta]$ ,  $x$  and  $y$  being the initial placement of the vehicle in the world coordinate system, and  $\theta$  being the initial vehicle heading in radians going clockwise from 0 to  $2\pi$ , 0 means the vehicle is pointing in the direction of the positive  $y$ -axis in case of a left-handed coordinate system,  $\pi/2$  indicates pointing in the positive  $x$ -axis and so forth.

The Khepera robot is controlled by the arrow keys on the keyboard. By using these keys and a couple of additional ones, all three types of basic movements described earlier can be accomplished. Here is a schematic schedule of the keys controlling the robot:

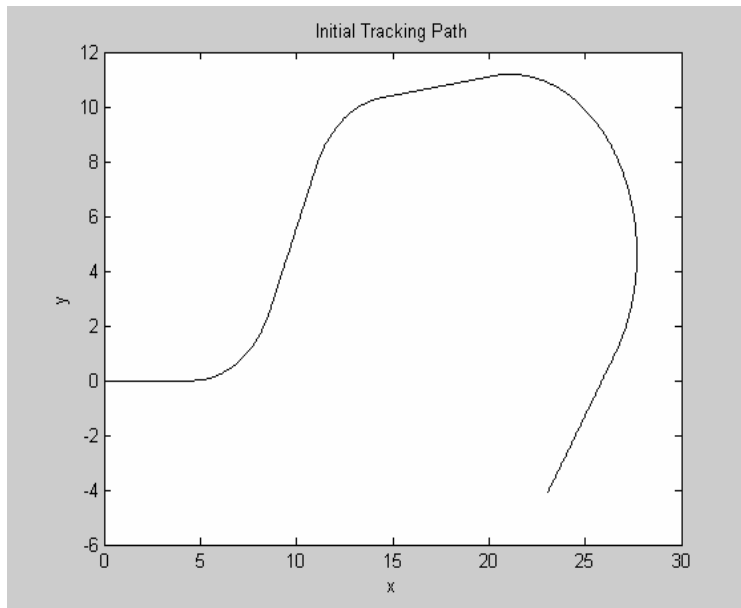
```
↑ (up-key)      = Move forward
↓ (down-key)    = Move Backwards
← (left-key)    = Rotate left
→ (right-key)   = Rotate right

↑ + ← = Turn soft left           (Turning radius = 6.5 cm)
↑ + → = Turn soft right

↑ + ← + ctrl = Turn sharp left   (Turning radius = 4.4 cm)
↑ + → + ctrl = Turn sharp right

shift = End path recording
```

Figure 12 illustrates an example of how a path may look like having been recorded by the remote unit. A graphics window showing the recorded path will always be displayed right after the user has pressed the shift-key. Notice that the *startpos* vector in this example was  $[0, 0, \pi/2]$ . I think it is most common and convenient to use origo as the positional starting coordinates. Also notice the different curve types, three sharp turns followed by a soft turn at the end of the path.



**Fig 12.** Path example recorded by the remote unit

Since all coding was done in Matlab I wrote a Mex-file for communication with the C-programming language [7]. A Mex-file is a compilation that can be called from Matlab. I needed to write a Mex-file with C-syntax to directly be able to use the appropriate Windows commands for sensing if any keys on the keyboard are currently being pressed down. The *GetAsyncKeyState* Windows function determines whether a key is up or down at the time the function is called [12]. Since it is a Windows function the header file *Windows.h* has to be included in order to get it to work. This function takes as argument a virtual key *vkey*, an id code representing the key to be checked. There are one unique virtual key code for each of the 256 ASCII-signs. These codes are written in the form *VK\_key*, where *key* is the unique id corresponding to a specific key. The *GetAsyncKeyState* function returns a negative number if being currently down, making it an easy task to implement the remote unit. For example, to check whether the up-arrow key is down at the moment one only have to write this simple line of C-code:

```
if (GetAsyncKeyState (VK_UP) < 0)
{
    /*Up-key is pressed*/
}
```

here *VK\_UP* is the virtual key code for the up-arrow key. Other keycodes are available at [12].

### 5.3 Obstacle Avoidance unit

The obstacle avoidance unit is a simple local implementation based entirely on proximity sensors. Initially I planned to implement some kind of global obstacle avoidance algorithm, like the Potential Field method or the Vector Field Histogram method. Global methods are generally much better than local ones, but instead they often require the use of more accurate

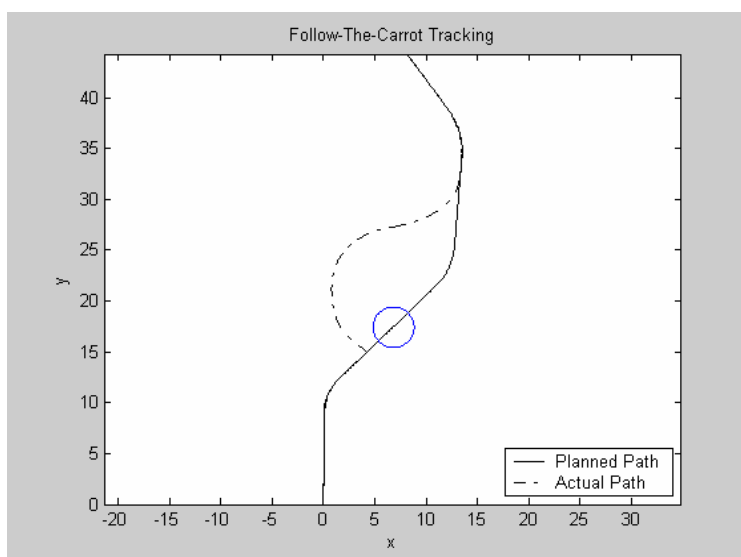
ultrasonic sensors for obstacle detection. Since the Khepera miniature robot isn't equipped with such advanced sensors, I decided that a local method would do in this project. One also has to consider that this project is a prework for the IFOR-project, involving autonomous forest machines, and as such only local obstacles appearing suddenly in front of the vehicle are of importance.

During the tracking phase this unit is responsible for repeatedly checking the four proximity sensors in the front. If any obstacles are detected the robot stops, rotates 90 degrees in some direction, and thereafter follows the edge of the obstacle until it points towards the current goal point again and no obstacle are in front of the robot. Thus the obstacle avoidance unit consists of three phases;

1. Stop
2. Rotate
3. Follow edge

A stop and rotate phase may seem unnecessary, but I saw no other possibility considering the bad accuracy of the proximity sensors. These sensors can only detect obstacles at a maximum distance of approximately three centimetres, so without these phases the robot would bump into the obstacle instead of avoiding it. The direction of rotation depends on a couple of factors in a decreasing level of priority. Firstly it depends on whether the robot confronts the obstacle to the left or to the right. If the obstacle is detected on the left front side, the direction of rotation will be to the right, and the other way around in case of the detection of an obstacle at the right front side. Secondly the direction of rotation depends solely on the location of the last point of the path. If the obstacle is placed directly in the forward direction, the robot will rotate in the direction of this last point. This behaviour avoids needless detours caused by the vehicles, and tries to optimize a minimum travel distance to the last goal point.

Detected obstacles are drawn in the shape of a circle of fixed size on the path tracking map. This procedure is not that accurate since obstacles can be of any shape and size, but at least it shows the user that an obstacle was found on the path and that the vehicle had to drive around it. Figure 13 shows an example of how it would look like on the graphics plot when obstacles are encountered.



**Fig 13.** Obstacle detected on path while tracking. Notice the robots reroute past the obstacle.



## 5.4 Path Tracking units

I have implemented and tested the follow-the-carrot method, the pure-pursuit method and the more recently developed vector pursuit method. The first two methods didn't cause much trouble at all, because the concepts behind the algorithms are very easy to understand thus making the implementation an easy task as well. The vector pursuit method however, was a bit more complicated to understand. This method required more time than the previous two together. Wit[3] proposes two different approaches one can use when implementing the vector pursuit path tracking algorithm. The big difference between these two methods is that the first approach initially ignores the nonholonomic constraints of the vehicle, while the second deals with these constraints directly. Nonholonomic constraints exist when the motion orthogonal to the vehicle's forward direction is not possible. Every vehicle with constraints in the ways they can move are nonholonomic. Parallel parking proves that cars exhibit this attribute as well, since steering angle constraints makes parking more difficult than it would have been if the wheels had been capable of turning perpendicular to the curb.

I implemented both of these approaches, but only the second method proved to be successful. The first method had a tendency to become unstable soon after starting the tracking procedure. In almost every type of test path, the robot began to oscillate about the path, and these oscillations became even more severe at higher speeds. I am not exactly sure of the reason for this behaviour, but I suspect it has to do with the initial ignoring of the nonholonomic constraints. Therefore I decided not to include test results from the first vector pursuit approach in this report. Method number two on the other hand proved to be very stable. No oscillations were noticed, and on first sight this method seemed as good as the pure pursuit method on simple test paths.

All the path tracking functions takes the same parameters as arguments, vector pursuit being the only exception as it also takes the k-factor as an argument. Otherwise the only arguments to the path tracking routines are; the recorded path to follow, the look-ahead distance, the inport used for communication with the robot or vehicle, the initial starting position and finally the speed of the vehicle. Here are the function headers of the implemented path tracking routines;

```
follow_the_carrot(inport,thepath,initial_pos,lookahead,speed)
pure_pursuit(inport,thepath,initial_pos,lookahead,speed)
vector_pursuit(inport,thepath,initial_pos,lookahead,kvalue,speed)
```

At the end of the tracking procedure, both the planned recorded path and the actual path taken by the robot is shown on the graphics display. The tracking errors are also calculated, giving the mean, maximum and standard deviation errors for position as well as heading. These errors are shown in the following order;

```
Mean =
      0.0189    0.0121

Max =
      0.0462    0.0759
```

Std =

0.129      0.0128

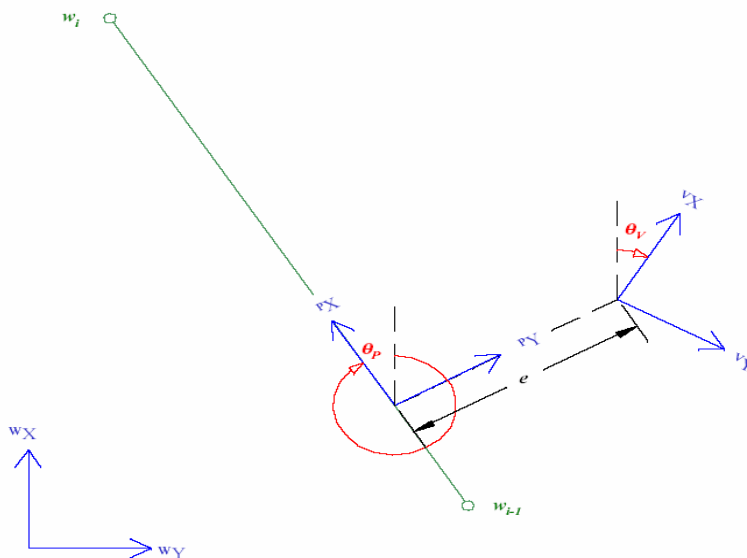
The first value on each line is the positional error, and the second is the heading error. In the next chapter I will explain the definition of the two different error types. Sometimes when testing the methods on a particular path, no visual difference can be seen from the graphics window, and in these kind of situations, the error measurements can be a good asset.

It is important to note that vector pursuit uses a right-handed coordinate system, while the others uses a left-handed coordinate system. This will not be so confusing when testing and running the algorithms though, the only visual difference is that the x-axis becomes the y-axis and vice versa. But at the implementation phase this caused a lot of problems, as I had to code separate conversion functions for either case or just integrate special conditions to handle both systems in one function.

## 6 Tests And Evaluations

### 6.1 Method for evaluating the algorithms

I have used two types of error measurements for evaluating the implemented path tracking methods – the position error and the heading error. These errors are calculated for every position reached by the robot during the tracking phase. The position error is defined to be the difference between the perpendicular projection of the robots position onto the path and the actual robot position. That is, all error measurements are done relative a new coordinate system with its origin at the projection point. Its x-axis is oriented in the path direction at this point, and its y-axis form a right-handed coordinate system. This perpendicular coordinate system is shown graphically in figure 14 below.



**Fig 14.** Perpendicular coordinate system used for error measurements and path tracking evaluation.

Thus the position error  $e$  is defined to be:

$$e = p_Y,$$

where  $p_Y$  is the y-coordinate of vehicles position in the perpendicular coordinate system. And the definition of the heading error is:

$$\theta_e = \theta_p - \theta_v,$$

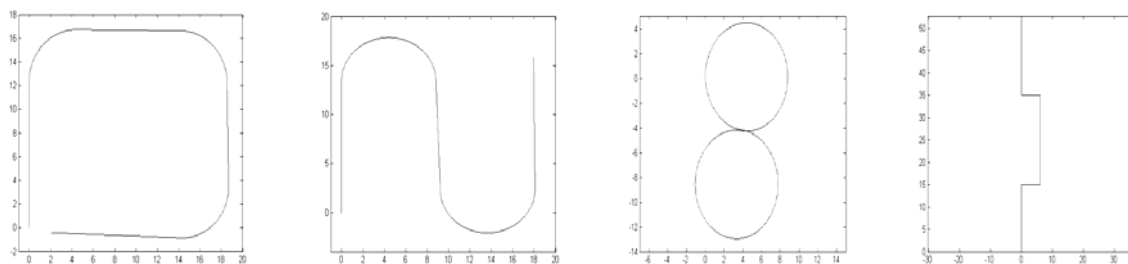
where  $\theta_p$  is the angle between the x-axis of the world coordinate system and the x-axis of the perpendicular coordinate system, and  $\theta_v$  is the angle from the x-axis of the world coordinate system to the x-axis of the vehicle coordinate system.

Based on the above described error measurements I have calculated the average and maximum positional and heading errors for the test paths, as well as the standard deviation error with regard to both position and heading.

## 6.2 Results

### 6.2.1 Survey

I have decided to test the path tracking methods on four different paths. These paths will each represent one difficulty that is common to the path following problem. The first path is a simple square, that shows each algorithms ability to go from a straight path to a sharp curve and then back to a straight section again. The second path is an s-curve, that is a curve shaped as the letter s but mirrored. The properties of this curve will show how each algorithm can cope with more sharp curves turning in both directions, both left and right. The third path is custom made to look like the figure eight, and here the big difficulty is coming from a left curve directly into a right curve and vice versa. Last but not least I will test the methods on a path containing a couple of really sharp turns or edges, to see at which level they can handle going from small errors in position and heading to really large ones. In reality this last test can simulate sudden obstacles appearing in front of the vehicle, causing it to move around the obstacle, or it could simulate noise received when estimating the current position of the vehicle, that will give rise to large sudden errors in position. The following chapter contains a selection of the test results of these paths. See Appendix A for the rest of the test results.



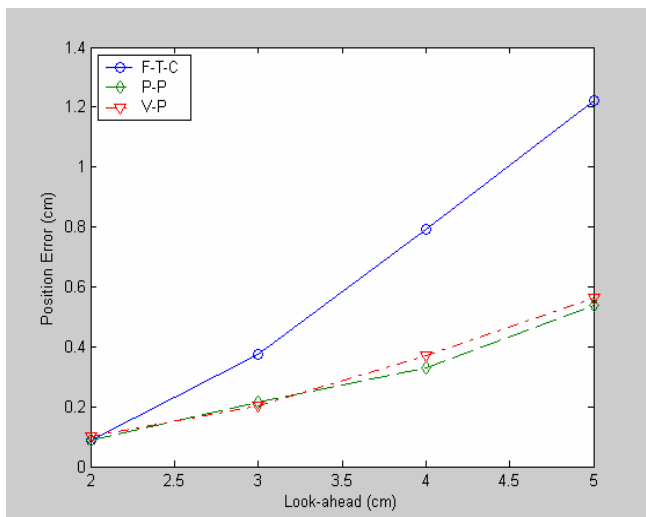
**Fig 15.** The four different test paths. Going from left to right we have the square, the s-curve, the figure 8 and the sharp edges paths.

A very important factor when testing these algorithms is the look-ahead distance  $L$ . After some initial testing it became clear that  $L$ -values smaller than 2 made all tracking methods rather unstable. I further noticed that values larger than five centimetres didn't contribute much to the evaluation, so for every test that follows I have used a look-ahead distance of 2,3,4 and 5 respectively. Although larger  $L$ -values could be used and still give satisfactory results, but then all test paths would have to be longer. The speed of the vehicle or robot is also an important issue. When testing some of the methods any potential differences in accuracy between them will only be apparent at larger speeds. So I looked at the tracking behaviour at 5.6 cm/s and 10.4 cm/s respectively. Here the decimal numbers comes of robot specific speed settings used for the Khepera, each unit being 0.8 cm/s. Only one exception was made, I changed the speed to 13.6 cm/s on the last test path containing sharp edges to further boost any potential differences amongst the methods.

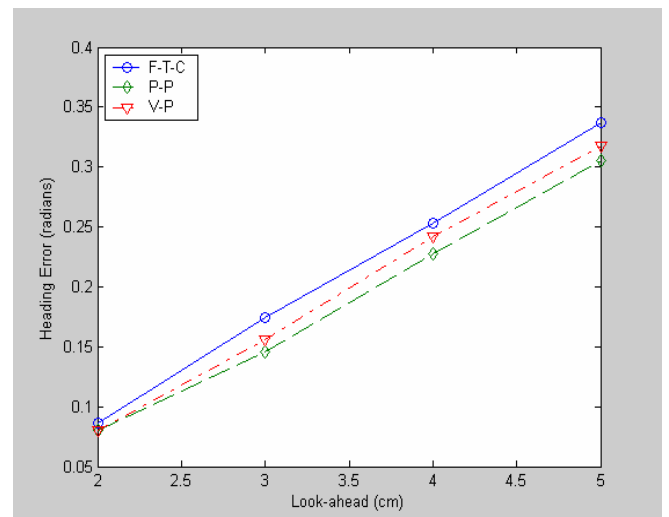
There is yet another variable that must be specified by the user when testing the vector pursuit algorithm, and this variable is called the  $k$ -factor [3]. This is a weighting factor that indicates how much of the calculated screw that is influenced by pure translation motion or pure rotational motion. The value of the  $k$ -factor was chosen through some experimental testing to be 20.

## 6.2.2 Evaluation

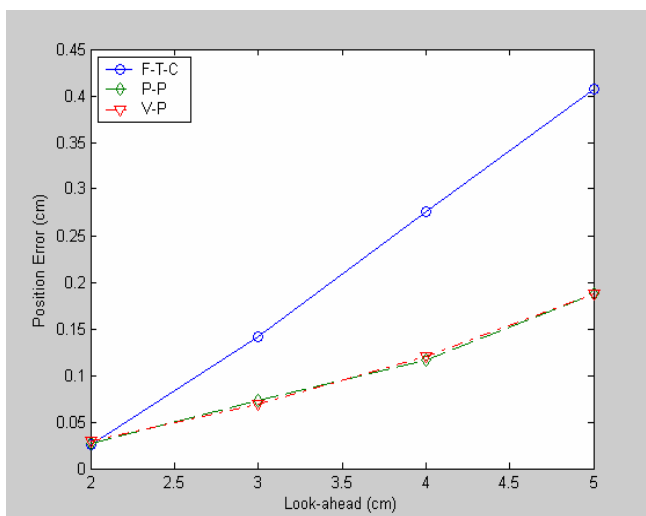
I started by testing each method on a path shaped as a square but with circular corners. Figure 16 through 19 shows the maximum and standard deviation error results with regard to both position and heading, for the methods compared to each other. Not surprisingly it turns out that the follow-the-carrot algorithm exhibits larger error values overall than the other two methods. The graphs also shows smaller differences in heading than in position on this specific path. This is both rather expected results, as the path doesn't offer any sharp turns, and moreover these turns are in one direction only. I have decided not to show all results done at the higher speed 10.4 cm/s, since they are very similar to the results at 5.6 cm/s. An interesting result comes from the standard deviation graphs, showing a big gap between follow-the-carrot and the other two methods. This points out that pure-pursuit and vector-pursuit is much less sensitive to small changes in the look-ahead distance than follow-the-carrot.



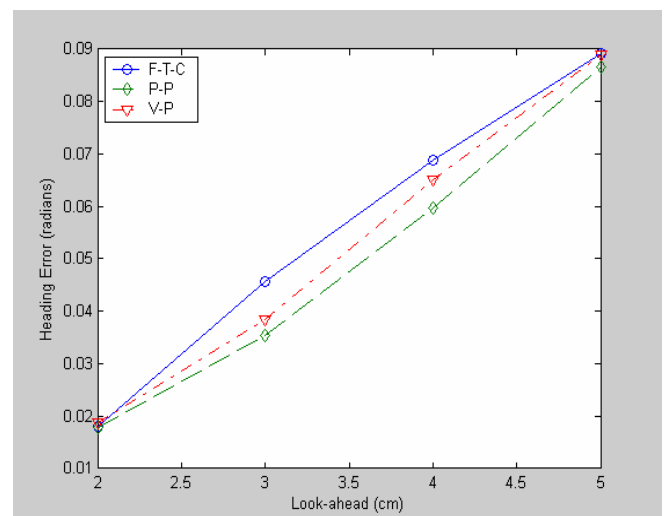
**Fig 16.** Max position error tracking a square path at 5.6 cm/s.



**Fig 17.** Max heading error tracking a square path at 5.6 cm/s.

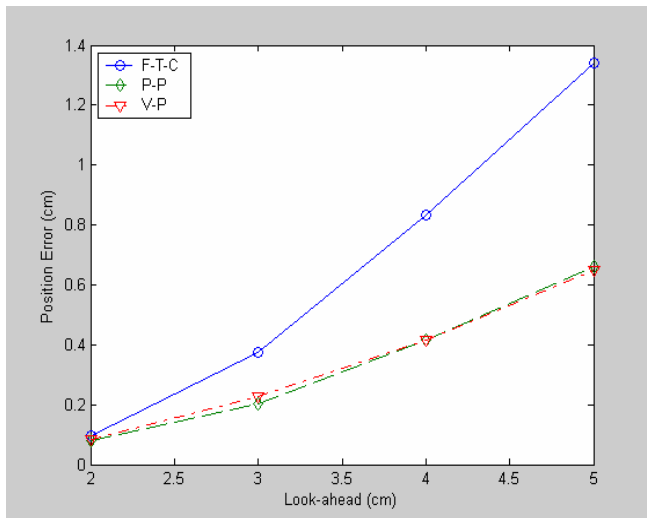


**Fig 18.** Standard deviation position error tracking a square path at 5.6 cm/s.

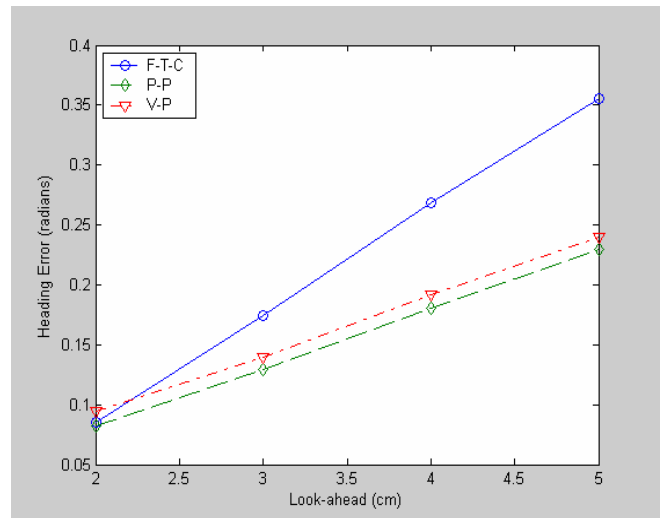


**Fig 19.** Standard deviation heading error tracking a square path at 5.6 cm/s.

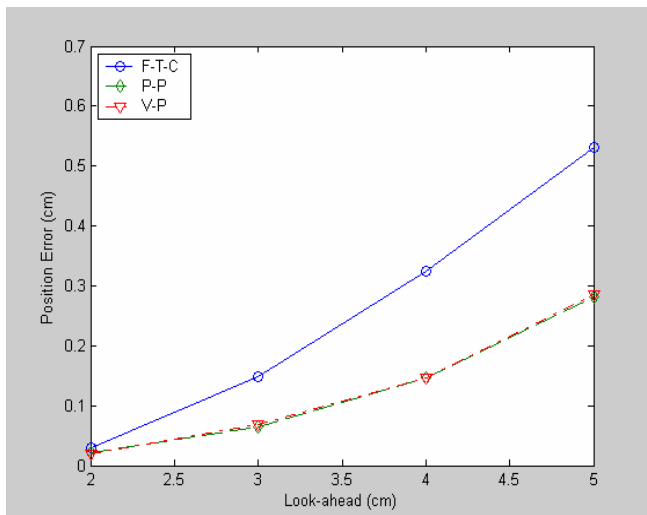
The tests performed on the S-shaped path showed pretty similar results, although this time the differences proved to be more obvious, especially with regard to the heading. This is all according to what I had expected. An S-shaped path contains curves in both directions adding another level of difficulty compared to the first path tested. Moreover these curves are both sharper and longer than the ones in the square path earlier. Note that there is almost nothing separating the pure-pursuit path tracking algorithm and the vector-pursuit approach. I suspect this has to do with the fact that there are one important similarity between these methods. This similarity being that vector-pursuit uses a pure-pursuit method to calculate the desired radius of curvature whenever the current path point is a curve. The mean and maximum tests results on this path can be seen in figures 20 through 23 below. Appendix B contains a collection of sample plots corresponding to all test paths.



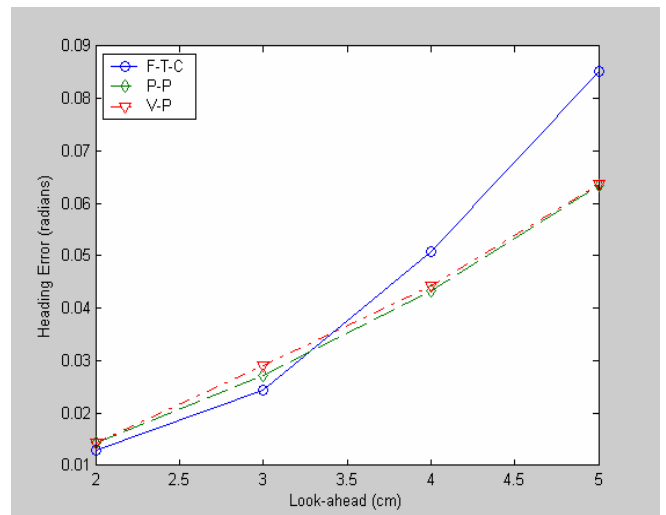
**Fig 20.** Max position error tracking an “S”-shape path at 5.6 cm/s.



**Fig 21.** Max heading error tracking an “S”-shape path at 5.6 cm/s.

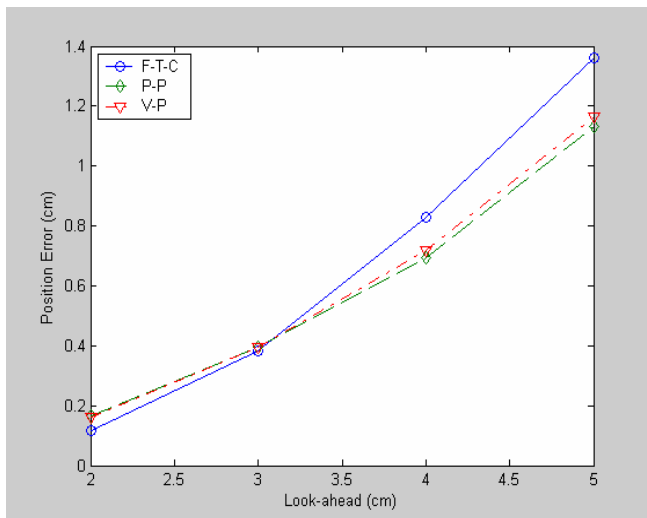


**Fig 22.** Mean position error tracking an “S”-shape path at 5.6 cm/s.

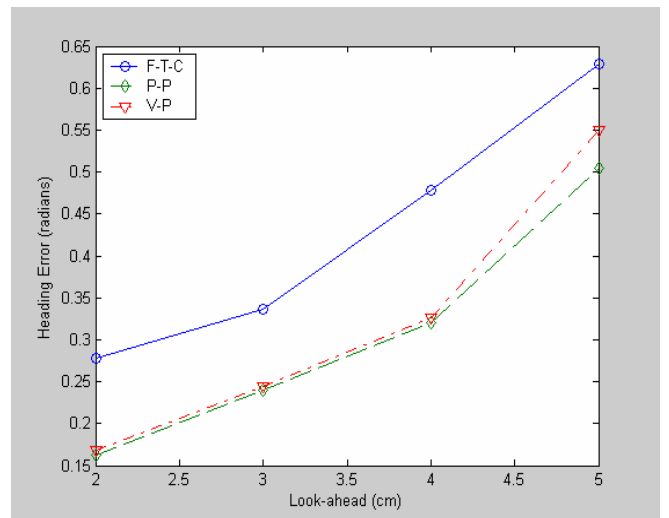


**Fig 23.** Mean heading error tracking an “S”-shape path at 5.6 cm/s.

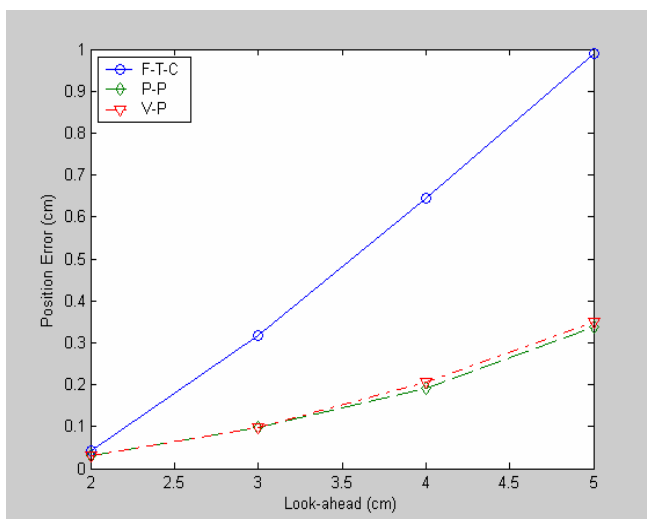
Then the path looking like the figure 8 was tested. As can be seen from the results in figures 24 through 27 below each technique was able to navigate the path with relatively small errors. The follow-the-carrot method still lacks in precision though, just as it did in the first two tests. Figures 24 through 27 shows the results of the path tracking techniques at 5.6 and at 10.4 cm/s. It's interesting to see from figure 27 that at higher speeds follow-the-carrot needs larger look-ahead distances to be able to remain the same precision. However the other methods weren't affected at higher speeds at all. The big difficulty in this test was the adjustment in transition from a left turn directly into a right turn and vice versa. At small look-ahead distances this transition was barely noticeable, but at larger values the problem becomes more evident. This shows on the side effects of using look-ahead distances greater than 0, which means the method is always going to start turning in advance, so consequently it will be cutting corners.



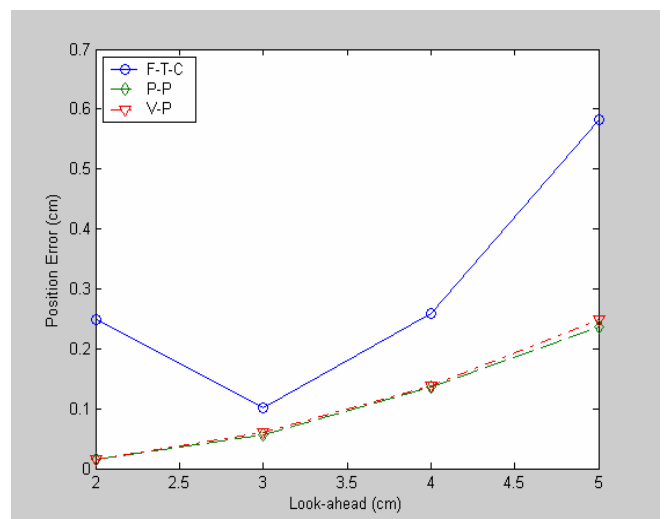
**Fig 24.** Max position error tracking an “8”-shape path at 5.6 cm/s.



**Fig 25.** Max heading error tracking an “8”-shape path at 5.6 cm/s.

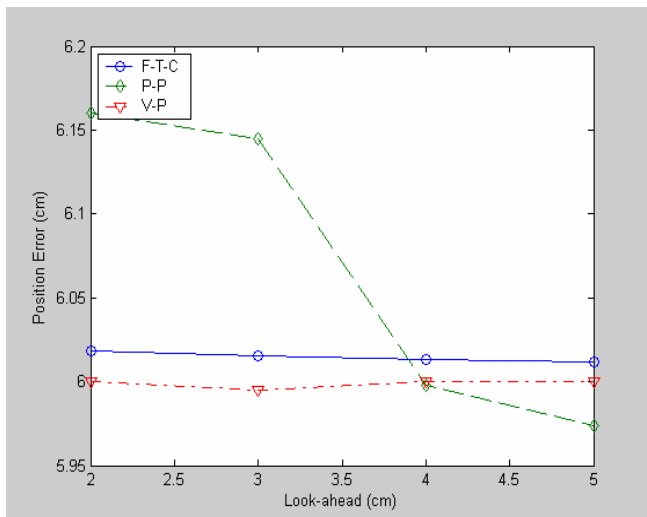


**Fig 26.** Mean position error tracking an “8”-shape path at 5.6 cm/s.

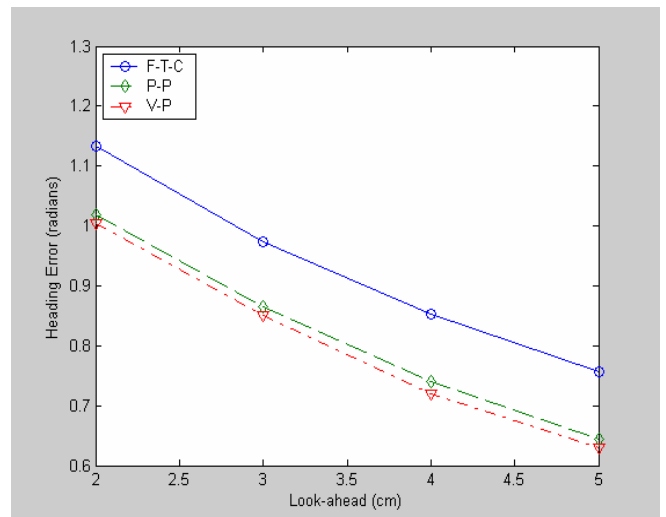


**Fig 27.** Mean position error tracking an “8”-shape path at 10.4 cm/s.

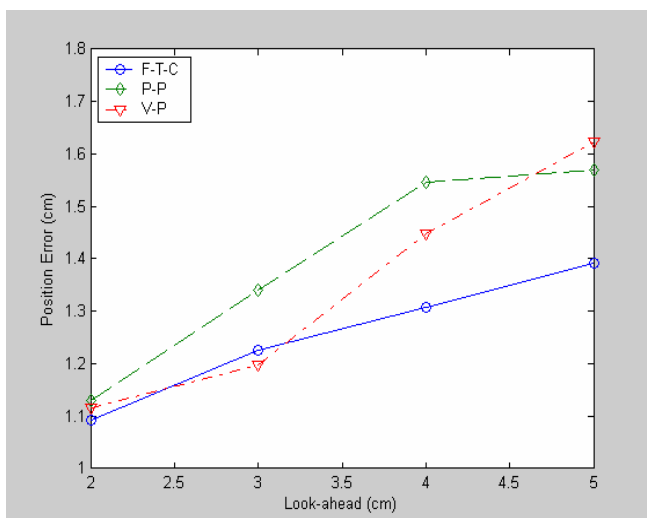
The last path used for testing the different techniques was a path containing a section that has been displaced 6 cm sideways to the right. The reason for this is to try to simulate going from a small error in position and heading directly to very large errors. Figures 28 through 31 shows how the tracking algorithms managed to handle this path. Vector-pursuit was the winner on this test, proving to have the smallest positional and heading errors compared to the other techniques. Follow-the-carrot actually showed good position error results, yet rather bad heading error values. Pure-pursuit produced uncertain results, as it shouldn't be so bad at tracking this type of path. The heading-error values of figure 29 stabilize if  $L > 5$ .



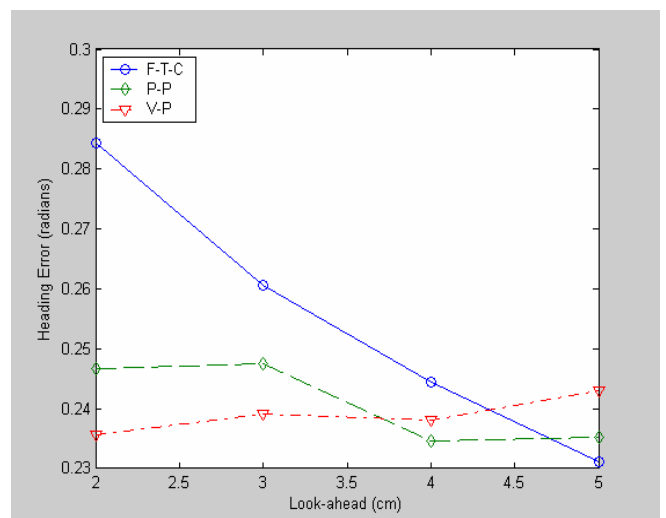
**Fig 28.** Max position error tracking a path with displacement at 13.6 cm/s.



**Fig 29.** Max heading error tracking a path with displacement at 13.6 cm/s.



**Fig 30.** Mean position error tracking a path with displacement at 13.6 cm/s.



**Fig 31.** Mean heading error tracking a path with displacement at 13.6 cm/s.

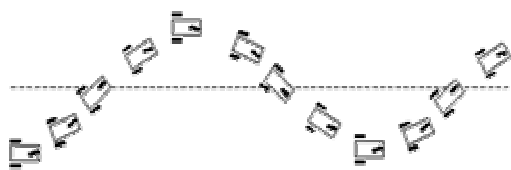
Note that the reason for the fact that follow-the-carrot gives better error results with regard to position is only because of a highly set proportional gain. This causes the method to turn more quickly when a sharp curve is encountered, thus reducing the position error slightly. The other techniques however results in more stable tracking procedures, on the cost of worse position error.



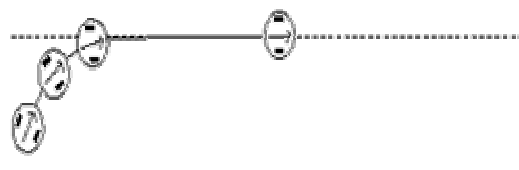
### 6.2.3 So why the small differences?

You may now be asking yourself why the differences between the path tracking methods described and tested in this report aren't more obvious than this. I think the reason for this can be found in the internal mechanics of the Khepera miniature robot. Both the pure-pursuit technique and the vector-pursuit method repeatedly calculates a turning radius that will move the tracking vehicle to the current goal point of the path. Based on this turning radius one can further calculate the appropriate wheel speeds that will correspond best to that current radius. However, these separate wheel speeds are always floating point numbers, like for example  $V_{\text{left}} = 2.3421$  and  $V_{\text{right}} = 6.7231$ . But the communication protocol controlling the robot knows only how to handle integers when setting the appropriate speed values. So the example speed above becomes  $V_{\text{left}} = 2$  and  $V_{\text{right}} = 7$  to the Khepera, therefore losing a bit of the tracking accuracy at every new calculation of turning radius. How much this will contribute to the small differences between the tracking techniques is hard to estimate though. I'm rather sure however that it will contribute to some extent, making it harder to evaluate each methods tracking accuracy.

Another reason could be the differential steering control used by the Khepera robot. Every tracking technique available has some kind of proportional control unit responsible for the steering of the vehicle. This unit takes as input either the heading error, as is the case for follow-the-carrot, or the turning radius as is the case for the other two methods. The steering system of the vehicle or robot decides the behaviour that results from the proportional control unit. A robot with car type steering behaves differently than a robot with differential steering. This is because with differential steering, your steering command directly controls the angle of the robot relative to the path. With the car steering robot, you are controlling the derivative of the angle of the robot. Then when trying to return to a line from some position in the near vicinity of this line, the car steered robot will oscillate back and forth while the differentially steered robot will converge smoothly without oscillation, that is if the speed of the robot is not too high.



**Fig 32.** Car steered robot



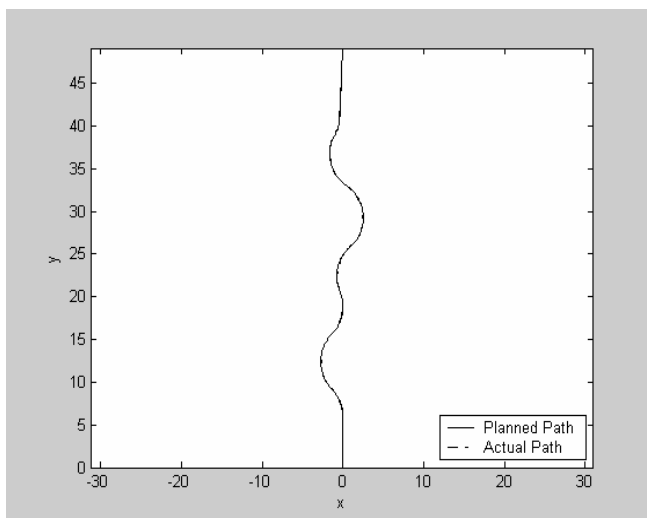
**Fig 33.** Differentially steered robot

This may also be a reason why the differences aren't that obvious. If the robot were to oscillate after a turn it would result in larger positional errors, as well as heading errors. On the other hand, in all the tests I performed it was only when testing the path containing a displacement that one method managed to oscillate a bit. The method that did this was of course follow-the-carrot, and this was noticeable only at very high speeds.

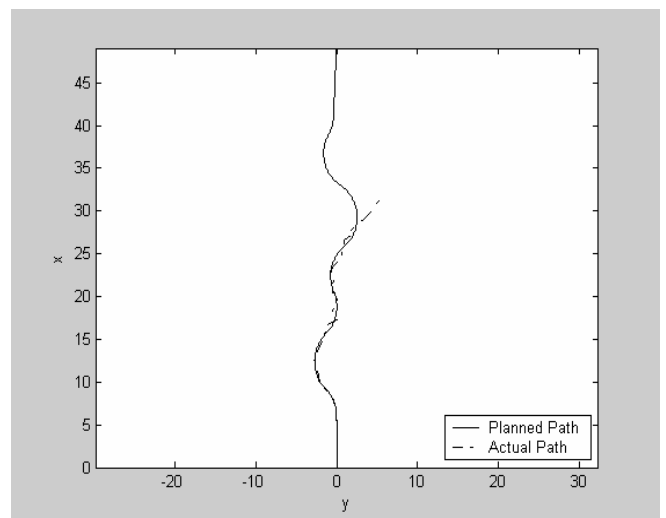
## 6.2.4 Pure-pursuit vs. Vector-pursuit

I also did some tests to try to figure out if there are any other differences between pure-pursuit and vector-pursuit besides the ones found on the previous tracking error evaluation. I wanted to know at what speeds and at what value of the k-factor that the vector-pursuit technique starts to oscillate and becomes unstable, and then compare this to the pure-pursuit method. For this purpose I created a really tricky course, with a lot of sharp turns going from one direction directly into the other, finishing off with a straight section at the end. Figures 34 through 38 shows some examples of how the techniques managed to handle this course. However any potential divergent behaviour will not be seen that clearly on the plots, the reason for this most likely being lack in plotting precision.

Initially I tested both methods on the path at 12 cm/s with a look-ahead distance of 2 cm. I didn't change the value of the k-factor, letting it remain at 20. Pure-pursuit proved to be very stable even at such a high speed, still tracking pretty smoothly considering the speed and the nature of the course. Vector-pursuit couldn't even finish the complete path, starting to oscillate at a high frequency when just having travelled halfway of the path..

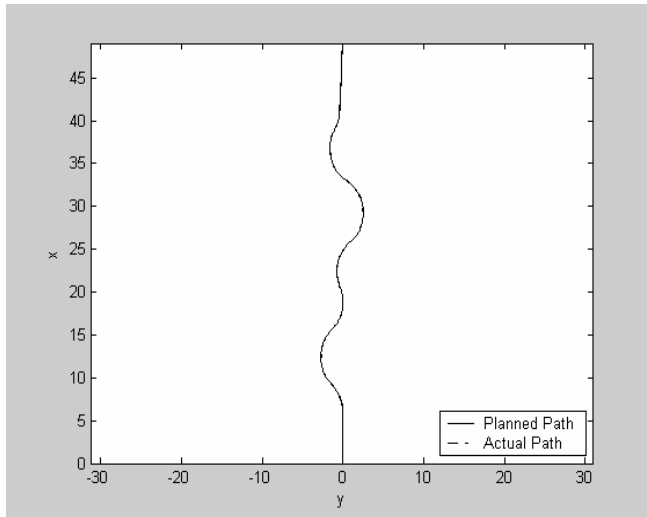


**Fig 34.** Pure-pursuit at 12 cm/s with a look-ahead of 2 cm



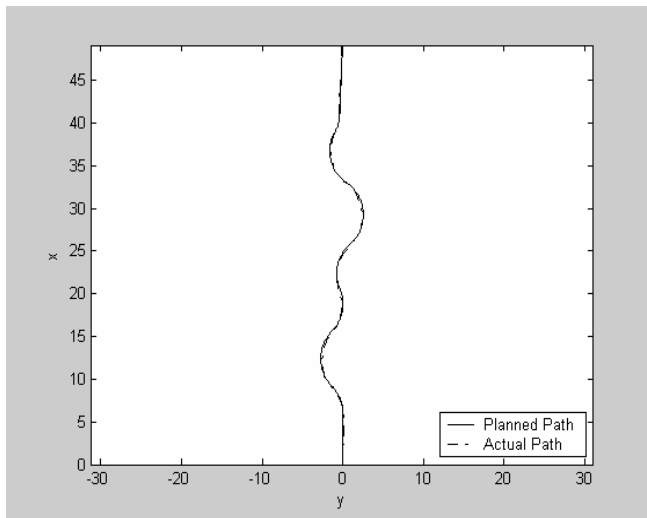
**Fig 35.** Vector-pursuit at 12 cm/s with a look-ahead of 2 cm and a k-value of 20

Considering the similarities between these techniques I found this behaviour rather strange, so I tried changing the value of the k-factor to a smaller number. Figure 36 shows the tracking procedure of vector-pursuit at the same speed but with a k-value of 5 instead of 20. Now the tracking remained smoothly from start to finish, as was the case for the pure-pursuit technique

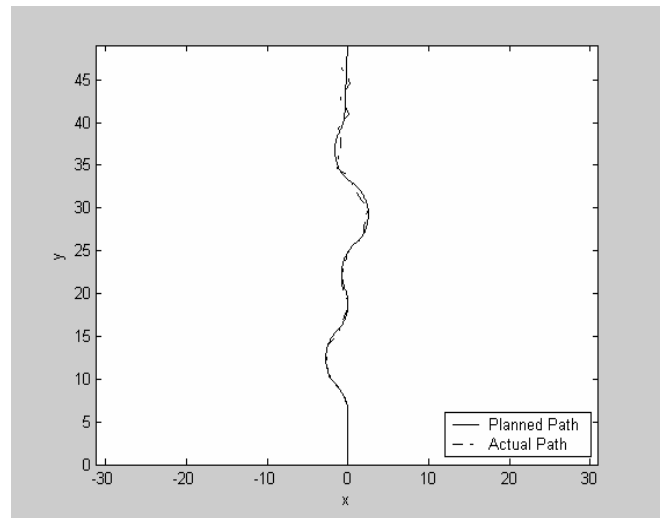


**Fig 36.** Vector-pursuit at 12 cm/s with a look-ahead of 2 cm and a k-value of 5

It seemed like vector-pursuit's tracking ability at high speeds was strictly dependent on the value of the k-factor. To confirm this I tested the two methods on the same path but at an even higher speed, 16 cm/s. Pure-pursuit was not able to track the path without oscillations. Vector-pursuit on the other hand had no problem at all, but the value of the k-factor had to be reduced to smaller number. The results are shown in figures 37 and 38. The unstable ride of the robot when using the pure-pursuit path tracking algorithm can be seen pretty clearly.



**Fig 37.** Vector-pursuit at 16 cm/s with a look-ahead of 2 cm and a k-value of 1



**Fig 38.** Pure-pursuit at 16 cm/s with a look-ahead of 2 cm

What conclusion can be drawn from this then? On normally stretched paths, that is on paths without heavy twists and turns, and at normal speeds, a pretty high k-value is required to get good tracking results. This is the reason I chose the value 20 on all previous tests made. On really tricky paths however, a smaller value has to be set in order to remain stability during tracking. As explained before the k-value is a ratio between translation and rotation, so depending on the nature of the path, deciding this value to an accurate level becomes an important task that can give vector-pursuit advantages over pure-pursuit in some cases. Pure-

pursuit is still better though at tracking normal paths at reasonable speeds, which is most often the case in real world applications.

### **6.3 Summary of results and conclusions**

As expected follow-the-carrot proved to be the worse overall tracking algorithm. On almost every test path used it produced higher error values with regard to both position and heading than the other two methods. This method was also the first to become rather unstable at higher speeds, making it useless for implementation in high velocity real world applications. However its simplicity with regard to both implementation and understanding of principles still makes it rather likeable. In cases such as for this project, when testing and comparing methods against each other is the primary issue, follow-the-carrot becomes important despite its drawbacks.

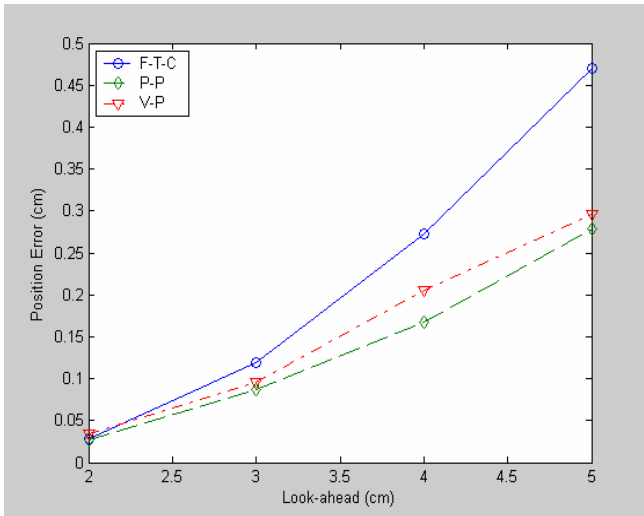
Pure-pursuit showed to be the best overall algorithm. On all the tests it produced far better tracking results than the follow-the-carrot approach, and it proved to be at least comparable to vector-pursuit on normal test paths at normal speeds. Moreover pure-pursuit was far easier to implement and understand than vector-pursuit, giving it another very important advantage that has to be considered before implementing a particular path tracking method. As explained in the previous chapter vector-pursuit was more stable in extreme situations, such as tracking a very tricky path with a lot of sharp turns at high speed. These situations don't occur that often in real applications though, making it an unnecessary effort to implement vector-pursuit when pure-pursuit is at least as good as the other and yet much easier to implement and understand.

One also has to consider factors like the type of steering used on the robot vehicle when testing the algorithms. Like I explained in chapter 6.2.3 these factors may contribute more or less to the results, and it is fully possible that vehicles using car-like steering produces different results in a similar project involving testing and comparing of path tracking algorithms. In fact this could be something to look into in future work, to examine any potential influences on tracking accuracy depending on vehicle steering type.

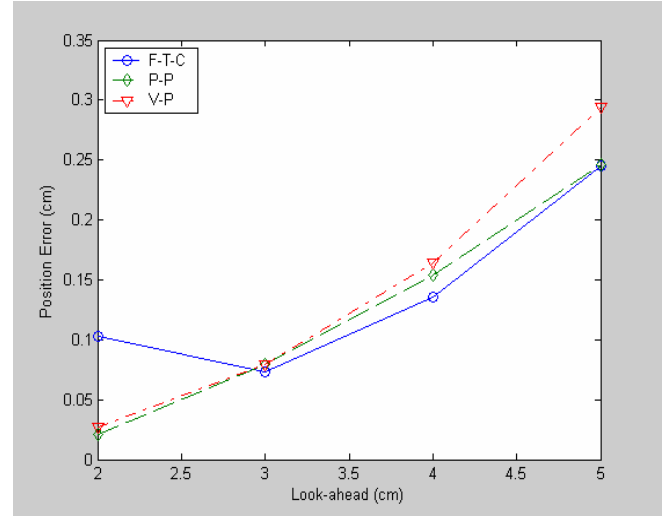
Another area of possible work is the problem of choosing an optimal look-ahead distance. I think the most common way of choosing this variable today is by simply testing some values and checking the tracking results for each value. There are however optimization techniques available, although telling how they would perform on these kind of algorithms is hard to do. There are many variables that affect the tracking performance together with the look-ahead distance, like the speed of the vehicle and the nature of the path to be followed. The k-value used in vector-pursuit may also be open for optimization, and would surely affect the overall performance of this method if this value could be optimized.

## 7 Appendix A

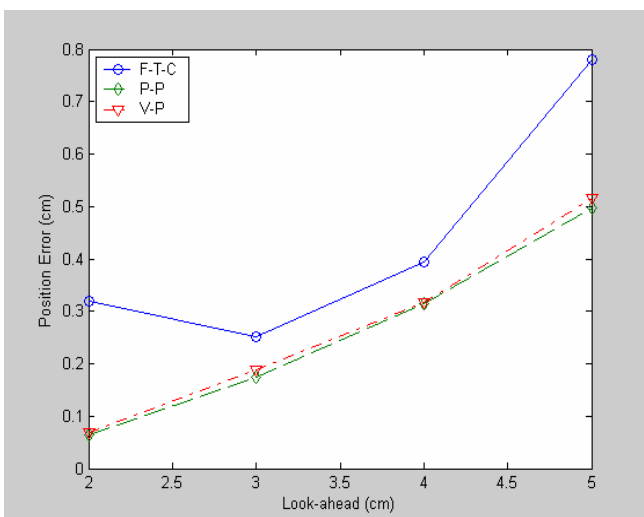
### 7.1 Simple Square



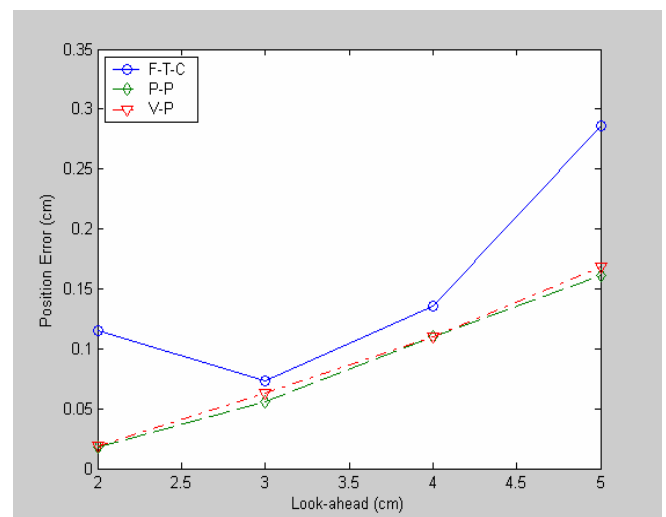
**Fig 39.** Mean position error tracking a square path at 5.6 cm/s.



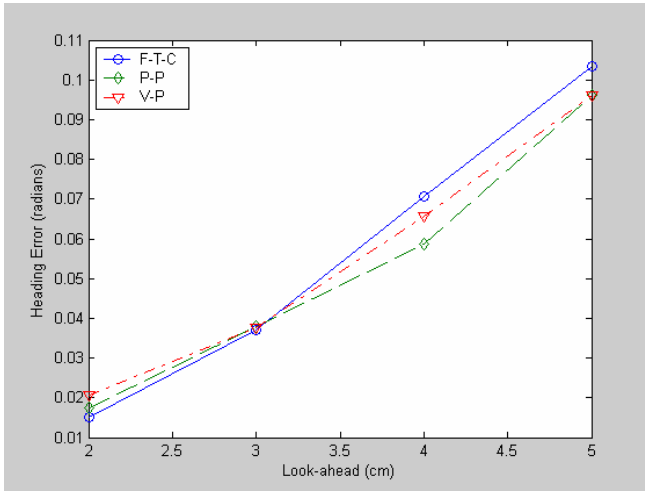
**Fig 40.** Mean position error tracking a square path at 10.4 cm/s.



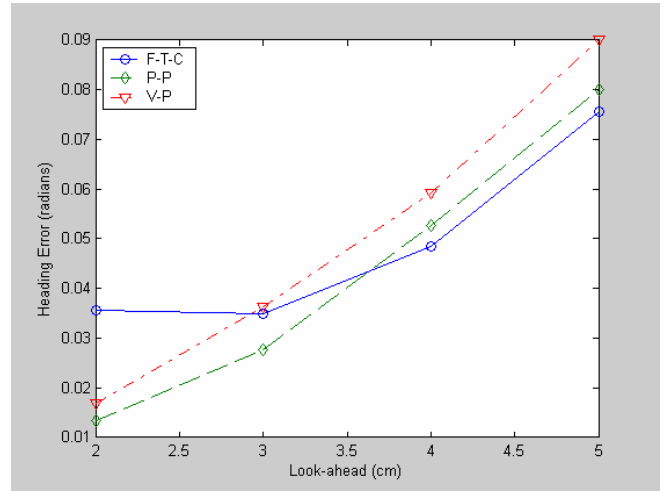
**Fig 41.** Max position error tracking a square path at 10.4 cm/s.



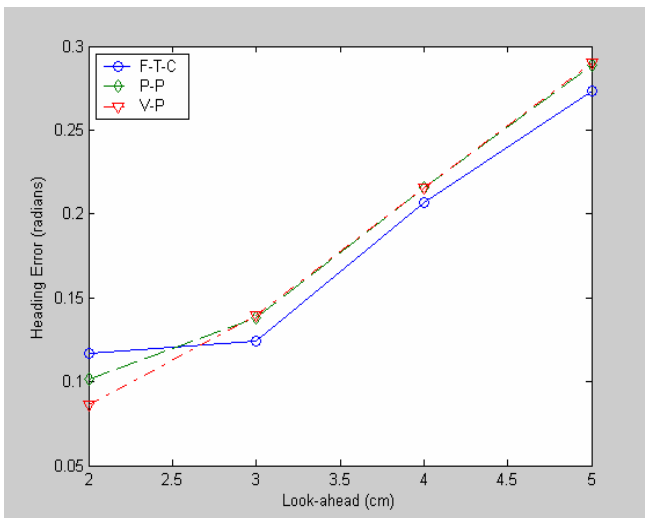
**Fig 42.** Standard deviation position error tracking a square path at 10.4 cm/s.



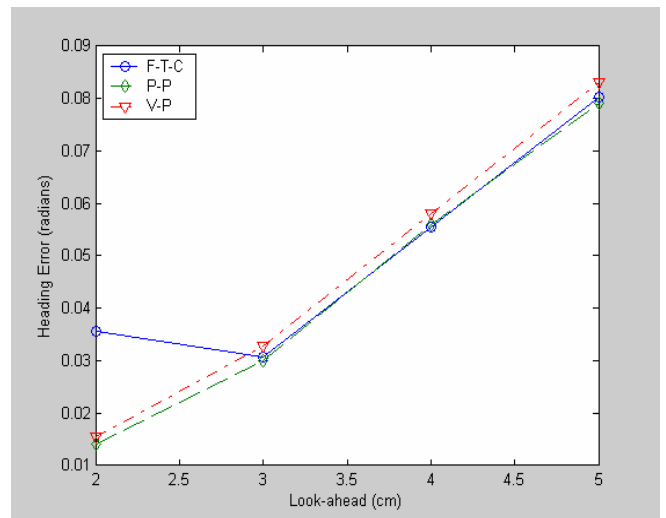
**Fig 43.** Mean heading error tracking a square path at 5.6 cm/s.



**Fig 44.** Mean heading error tracking a square path at 10.4 cm/s.

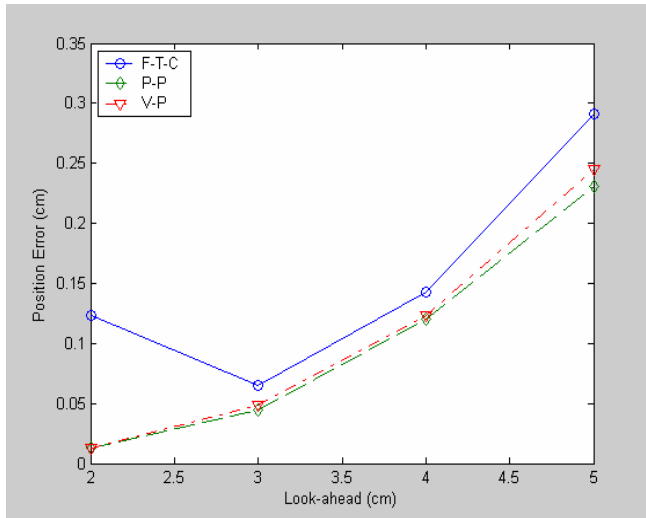


**Fig 45.** Max heading error tracking a square path at 10.4 cm/s.

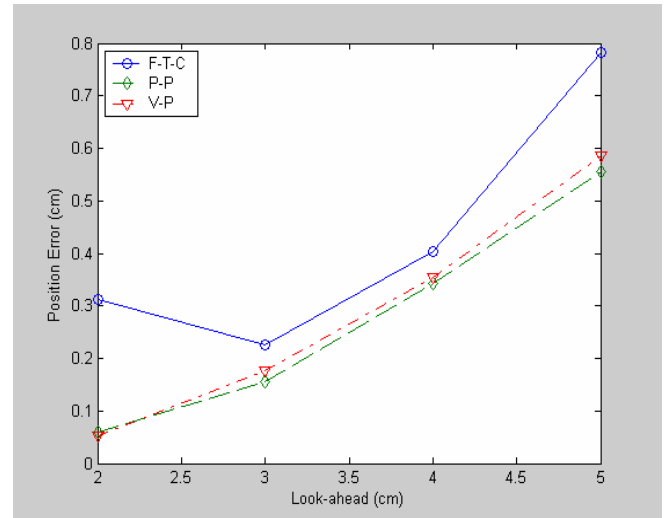


**Fig 46.** Standard deviation heading error tracking a square path at 10.4 cm/s.

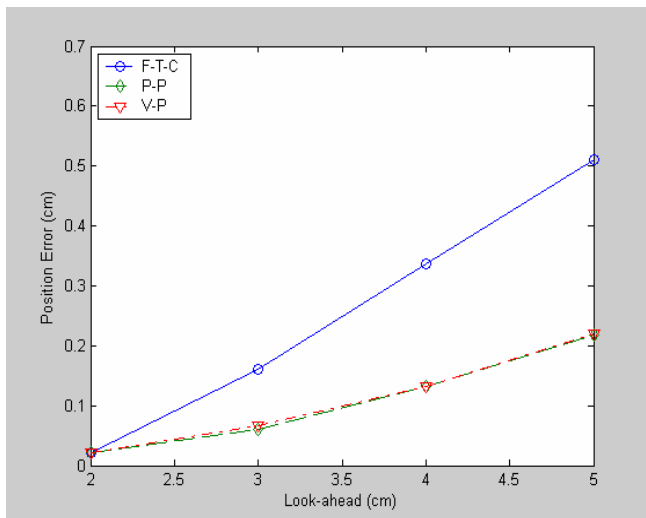
## 7.2 S-curve



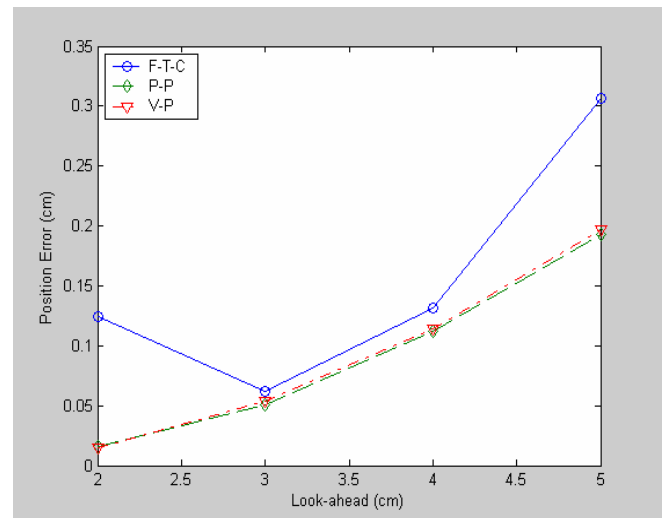
**Fig 47.** Mean position error tracking an "S"-shape path at 10.4 cm/s.



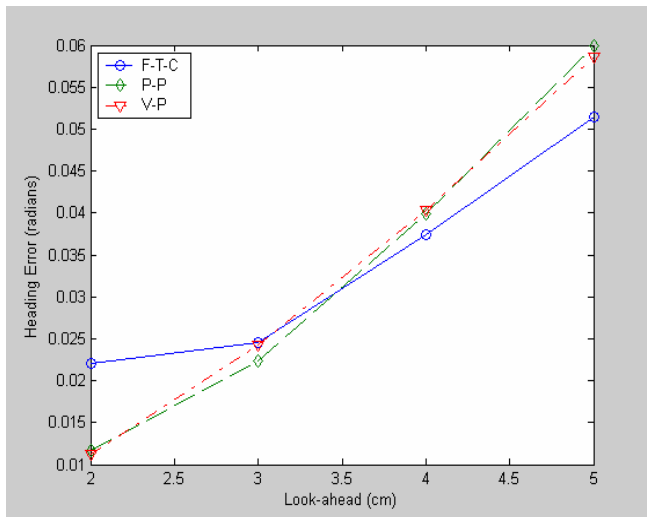
**Fig 48.** Max position error tracking an "S"-shape path at 10.4 cm/s.



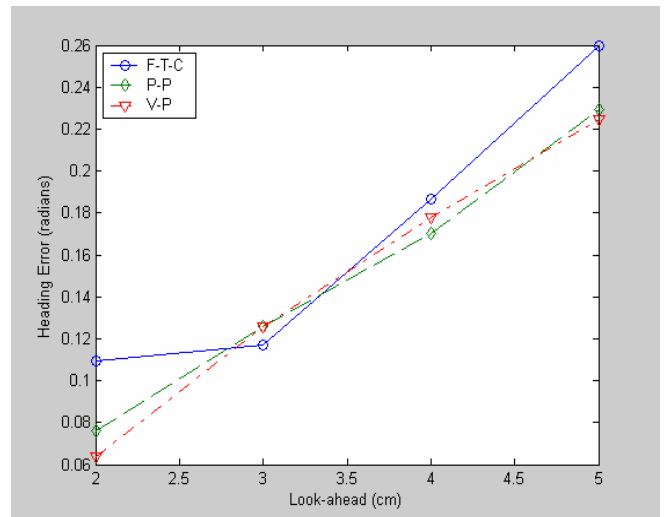
**Fig 49.** Standard deviation position error tracking an "S"-shape path at 5.6 cm/s.



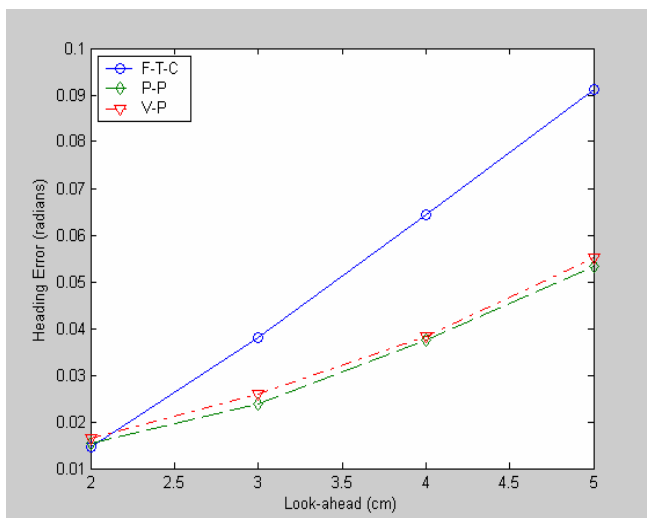
**Fig 50.** Standard deviation position error tracking an "S"-shape path at 10.4 cm/s.



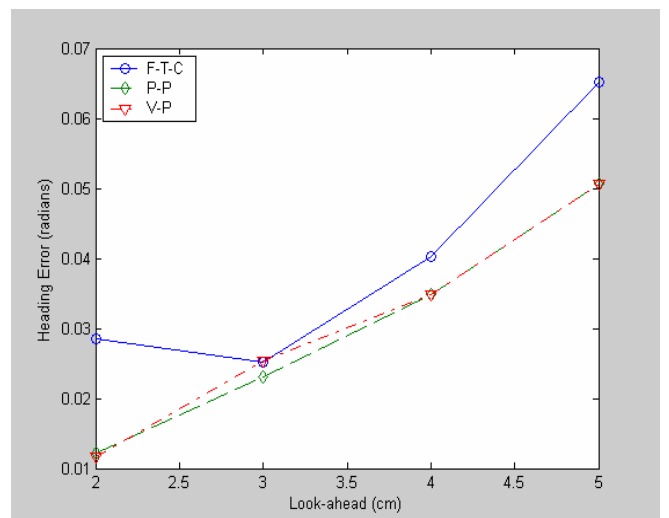
**Fig 51.** Mean heading error tracking an “S”-shape path at 10.4 cm/s.



**Fig 52.** Max heading error tracking an “S”-shape path at 10.4 cm/s.



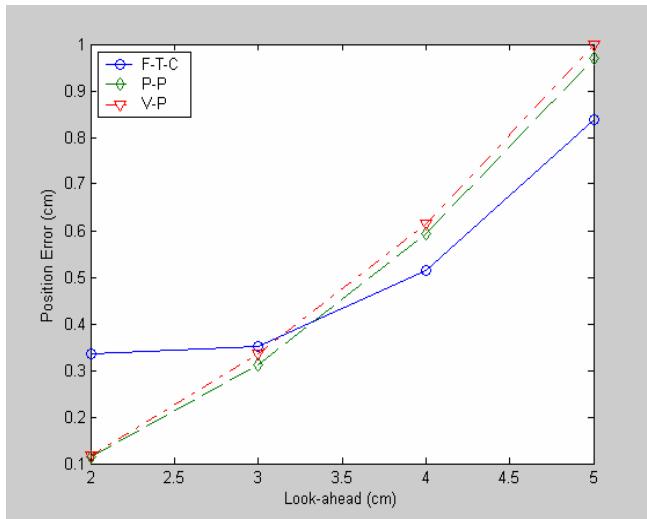
**Fig 53.** Standard deviation heading error tracking an “S”-shape path at 5.6 cm/s.



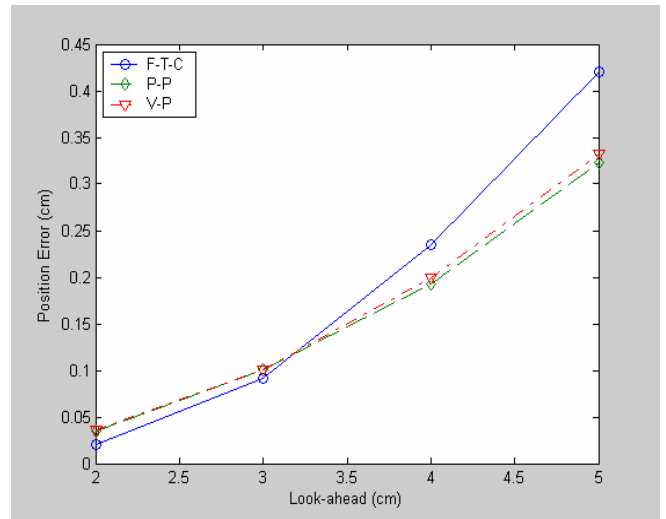
**Fig 54.** Standard deviation heading error tracking an “S”-shape path at 10.4 cm/s.



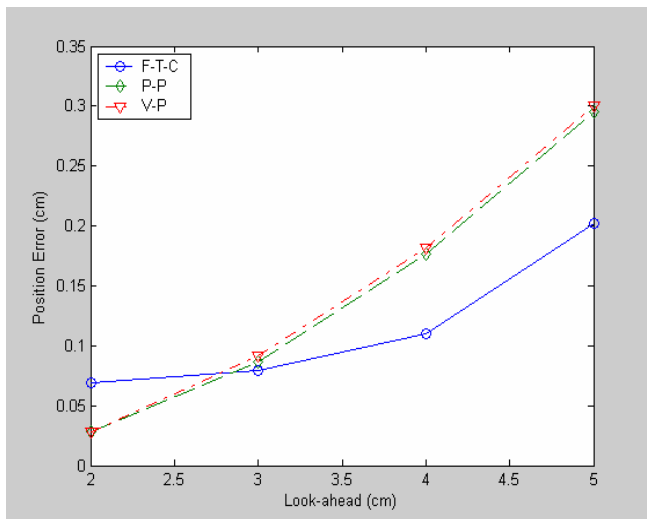
### 7.3 8-figure



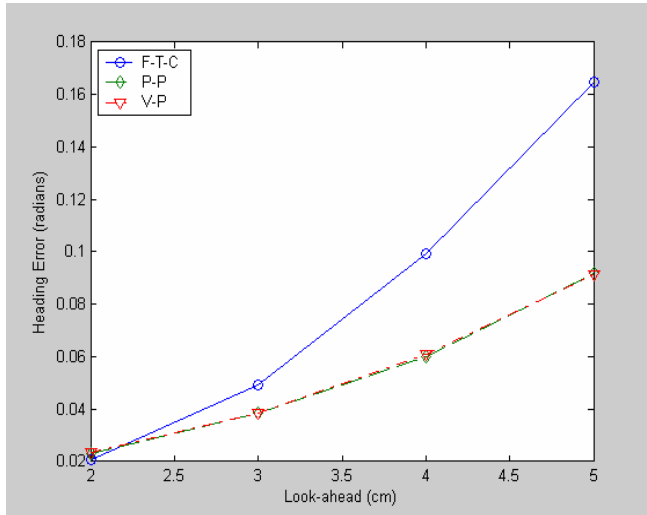
**Fig 55.** Max position error tracking an “8”-shape path at 10.4 cm/s.



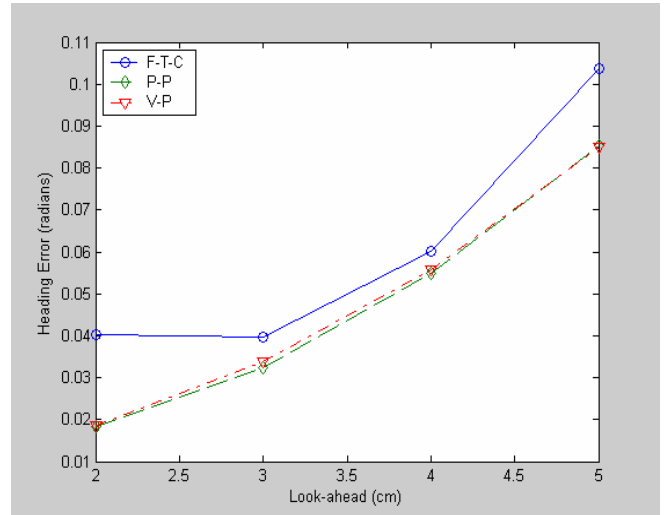
**Fig 56.** Standard deviation position error tracking an “8”-shape path at 5.6 cm/s.



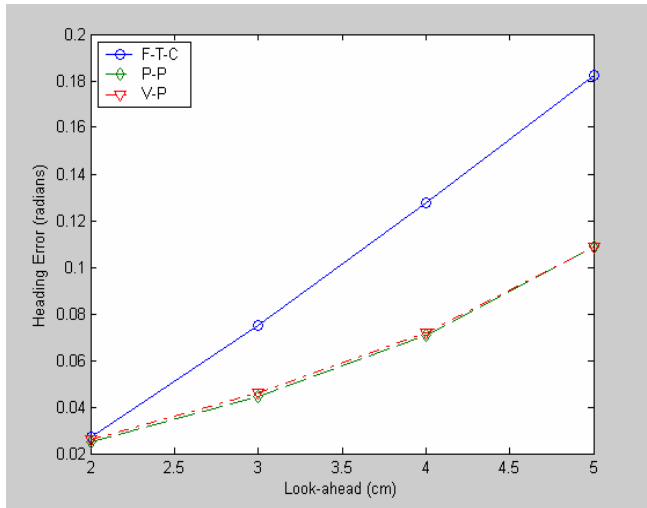
**Fig 57.** Standard deviation position error tracking an “8”-shape path at 10.4 cm/s.



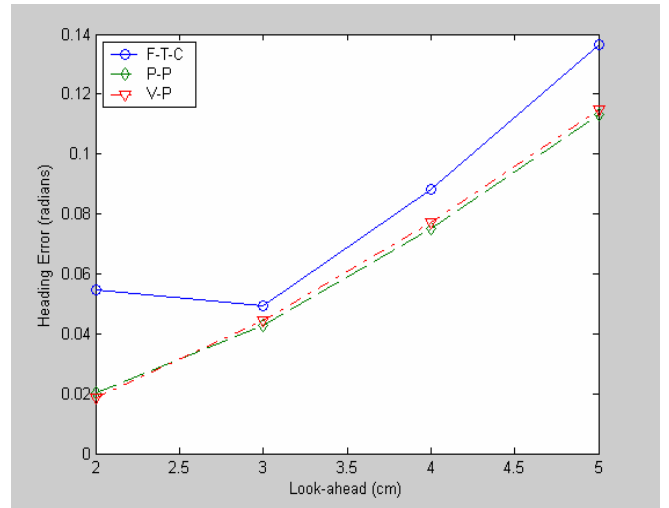
**Fig 58.** Mean heading error tracking an “8”-shape path at 5.6 cm/s.



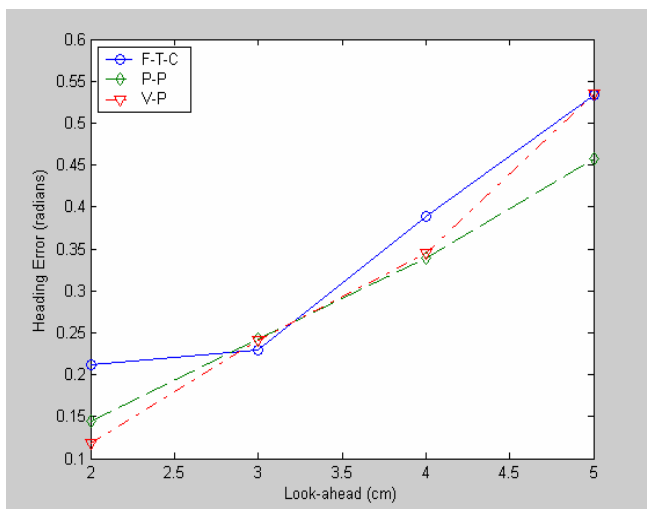
**Fig 59.** Mean heading error tracking an “8”-shape path at 10.4 cm/s.



**Fig 60.** Standard deviation heading error tracking an “8”-shape path at 5.6 cm/s.

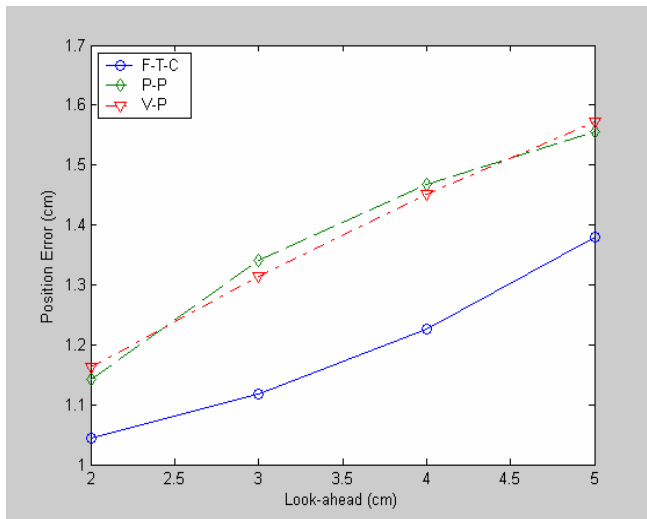


**Fig 61.** Standard deviation heading error tracking an “8”-shape path at 10.4cm/s.

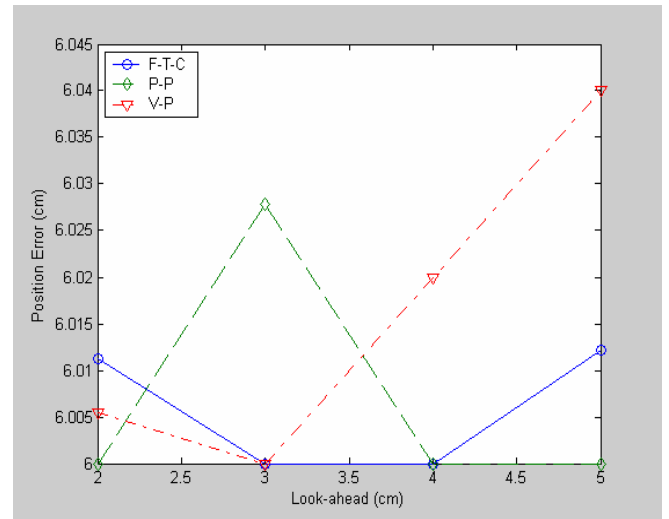


**Fig 62.** Max heading error tracking an “8”-shape path at 10.4 cm/s.

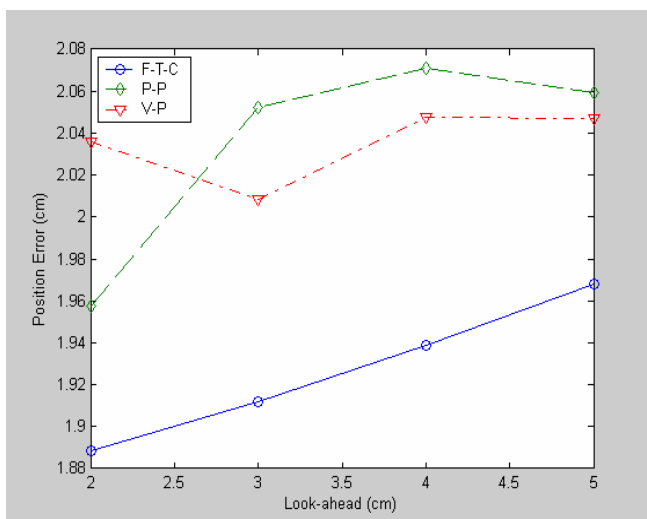
## 7.4 Path with a displacement section



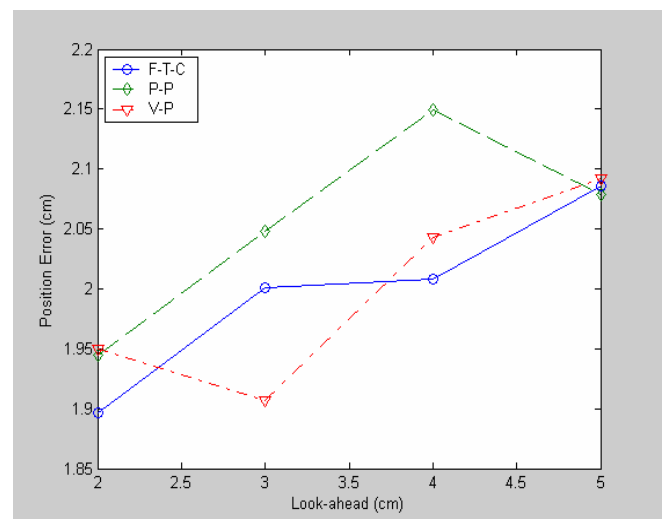
**Fig 63.** Mean position error tracking a path with displacement at 10.4 cm/s.



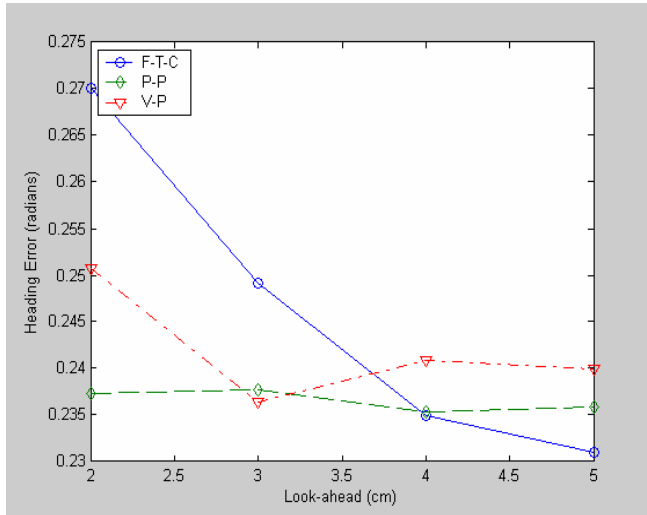
**Fig 64.** Max position error tracking a path with displacement at 10.4 cm/s.



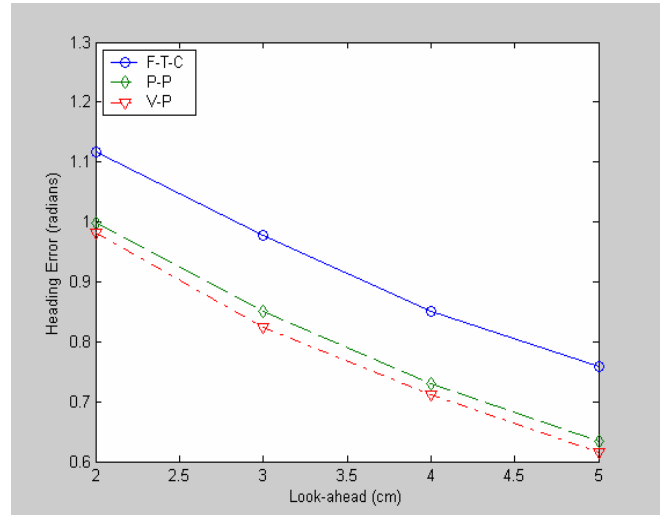
**Fig 65.** Standard deviation position error tracking a path with displacement at 10.4 cm/s.



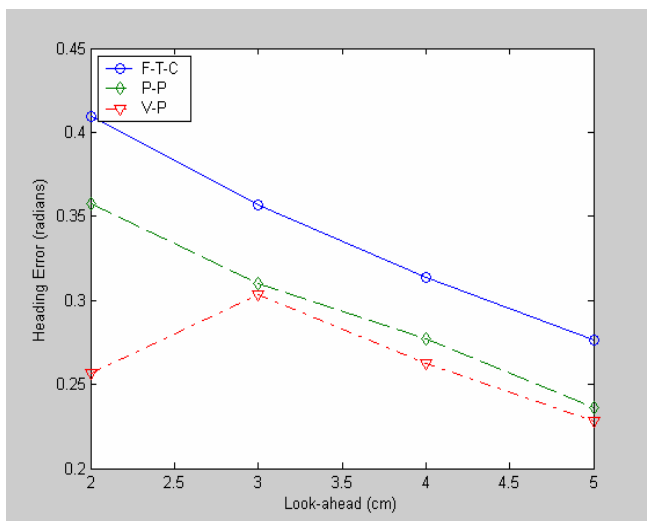
**Fig 66.** Standard deviation position error tracking a path with displacement at 13.6 cm/s.



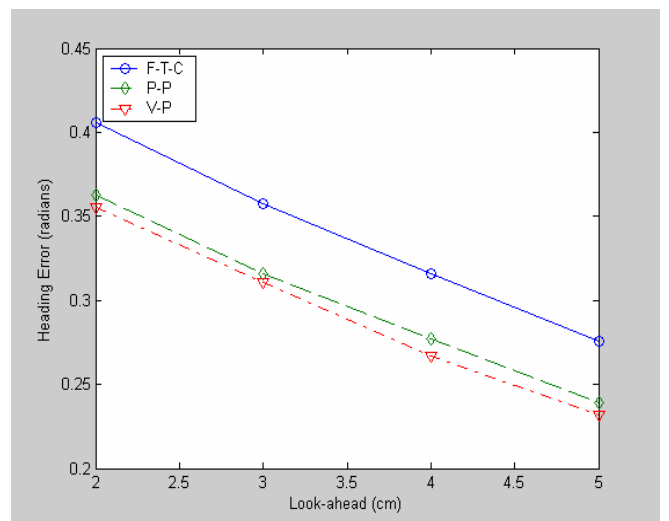
**Fig 67.** Mean heading error tracking a path with displacement at 10.4 cm/s.



**Fig 68.** Max heading error tracking a path with displacement at 10.4 cm/s.



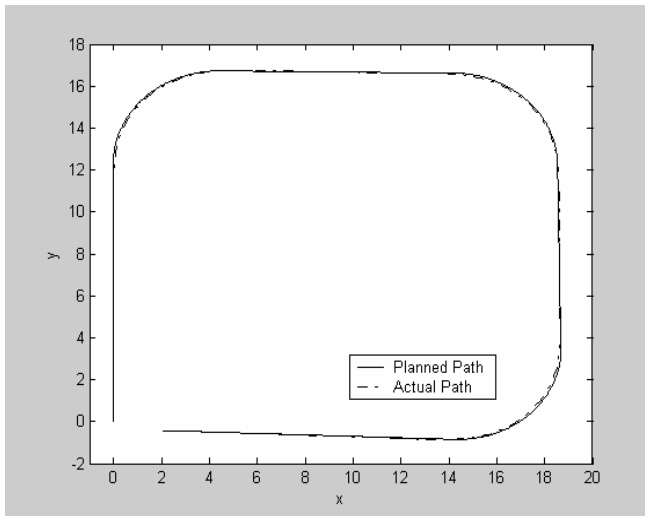
**Fig 69.** Standard deviation heading error tracking a path with displacement at 10.4 cm/s.



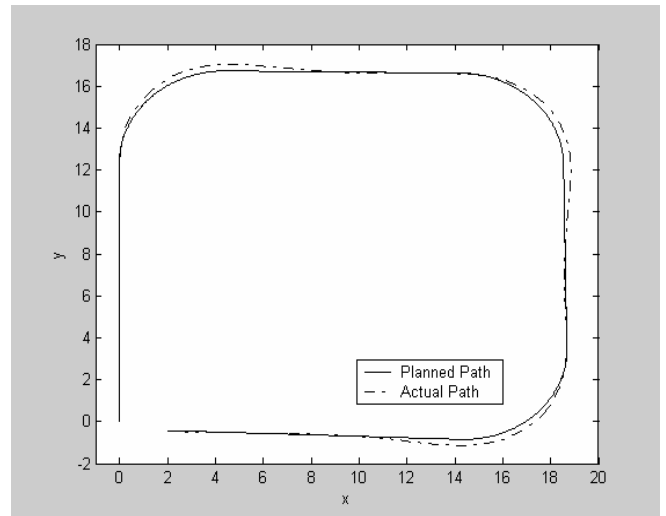
**Fig 70.** Standard deviation heading error tracking a path with displacement at 13.6 cm/s.

## 8 Appendix B

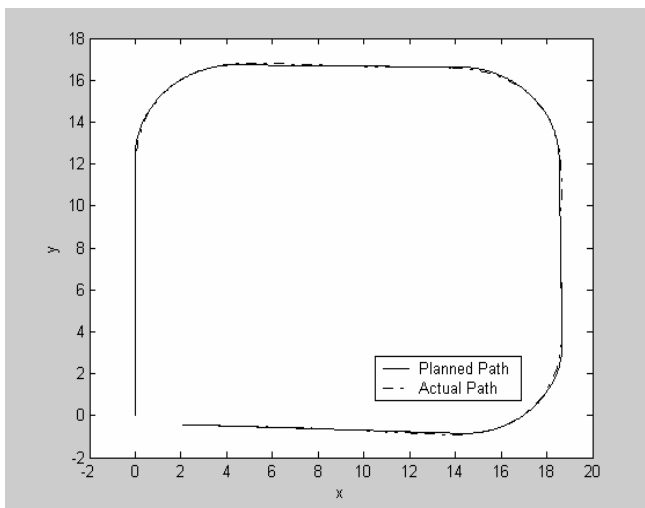
These are the plots that corresponds to all the test results in the previous chapter. They were created using Matlab 6.5 by first using the remote unit to create the path, and then executing each path tracking unit on this path while saving all the tracking coordinates. The planned path is the recorded route the robot is following, and the dotted path is the actual route taken during the tracking phase.



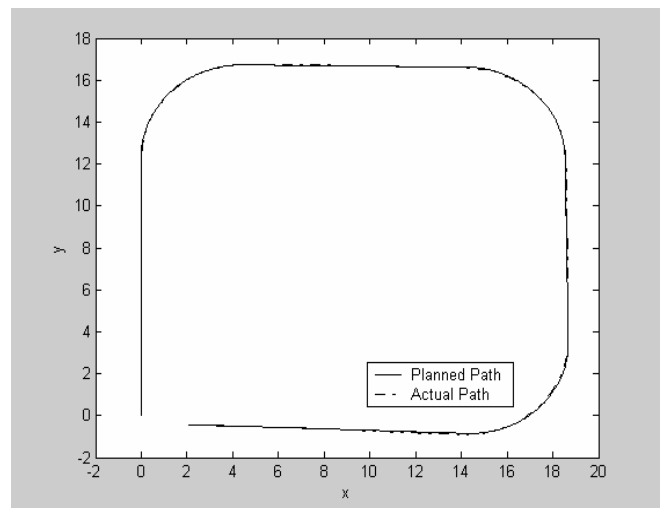
**Fig 71.** Follow-the-carrot at 5.6 cm/s with a look-ahead of 2 cm



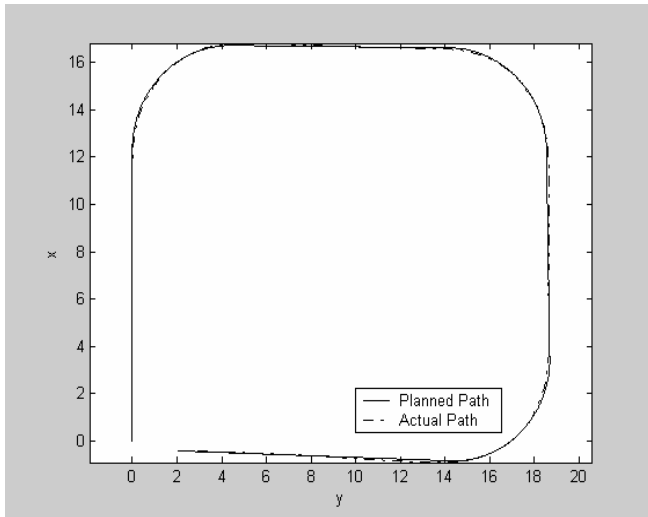
**Fig 72.** Follow-the-carrot at 10.4 cm/s with a look-ahead of 2 cm



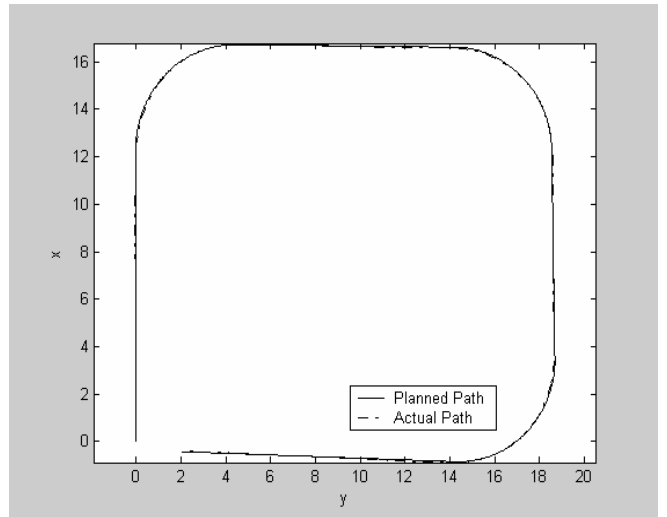
**Fig 73.** Pure-Pursuit at 5.6 cm/s with a look-ahead of 2 cm



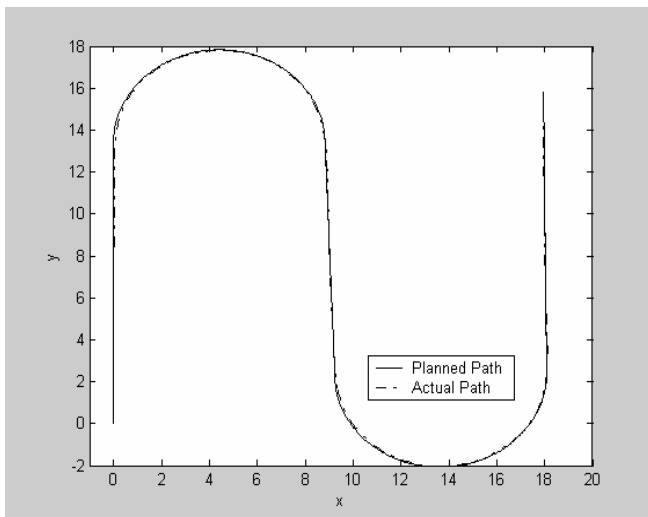
**Fig 74.** Pure-Pursuit at 10.4 cm/s with a look-ahead of 2 cm



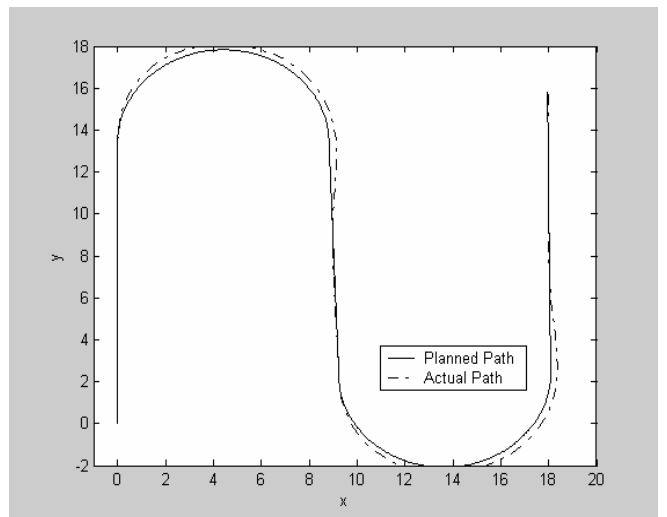
**Fig 75.** Vector-Pursuit at 5.6 cm/s with a look-ahead of 2 cm



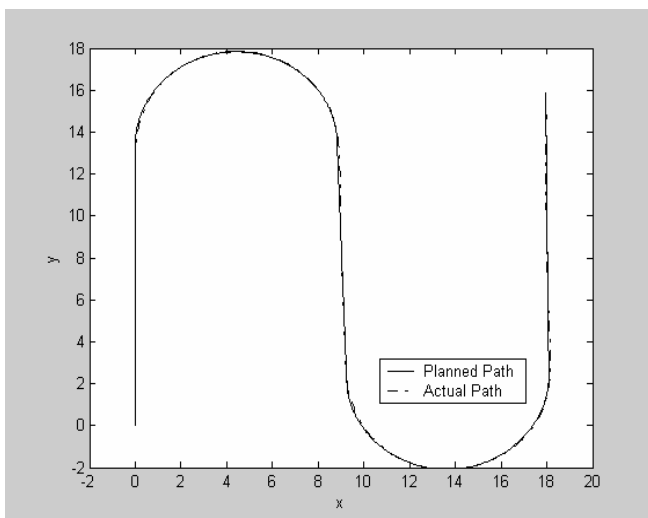
**Fig 76.** Vector-Pursuit at 10.4 cm/s with a look-ahead of 2 cm



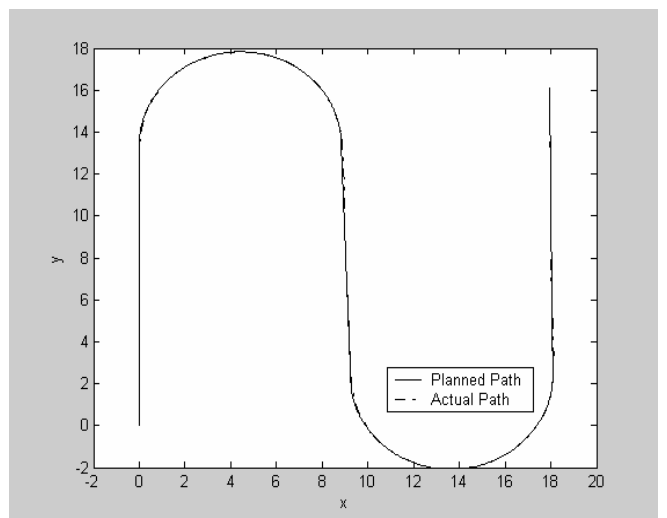
**Fig 77.** Follow-the-carrot at 5.6 cm/s with a look-ahead of 2 cm



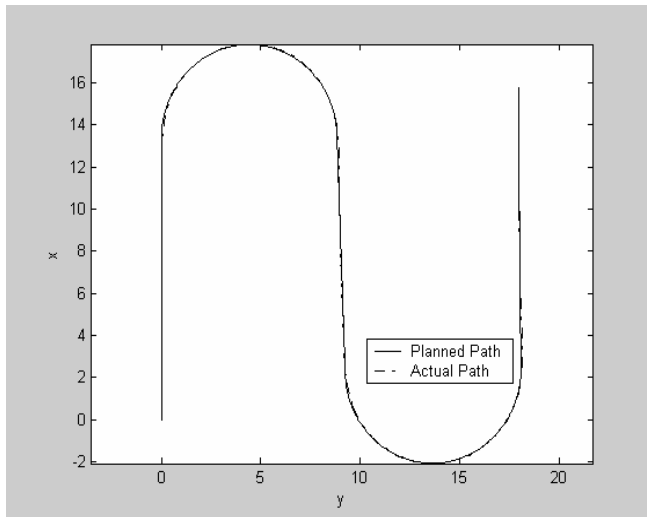
**Fig 78.** Follow-the-carrot at 10.4 cm/s with a look-ahead of 2 cm



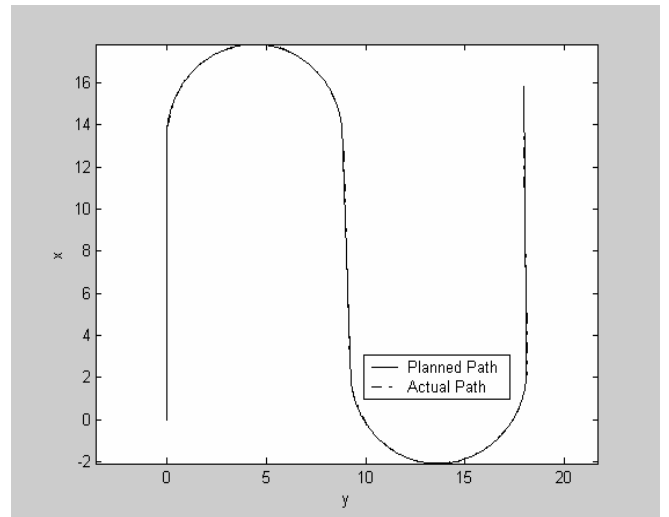
**Fig 79.** Pure-pursuit at 5.6 cm/s with a look-ahead of 2 cm



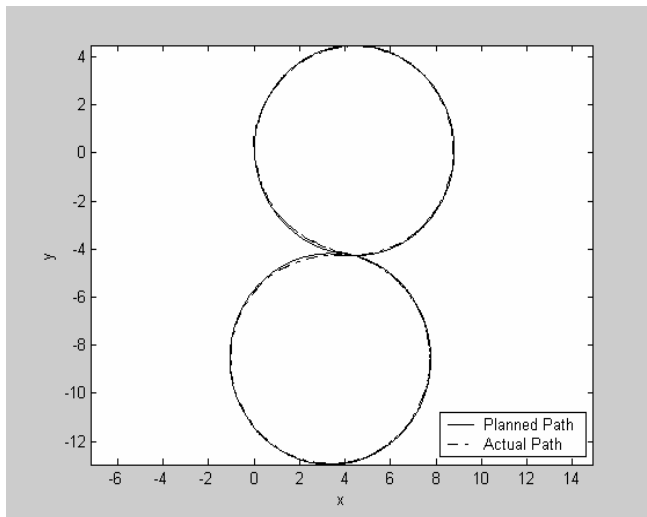
**Fig 80.** Pure-pursuit at 10.4 cm/s with a look-ahead of 2 cm



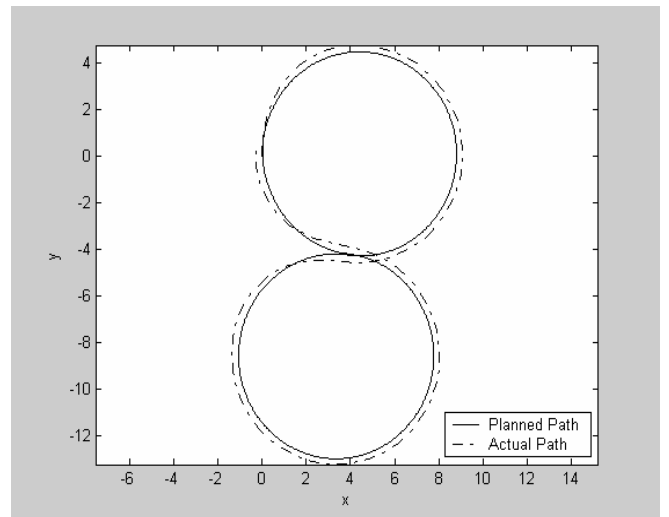
**Fig 81.** Vector-pursuit at 5.6 cm/s with a look-ahead of 2 cm



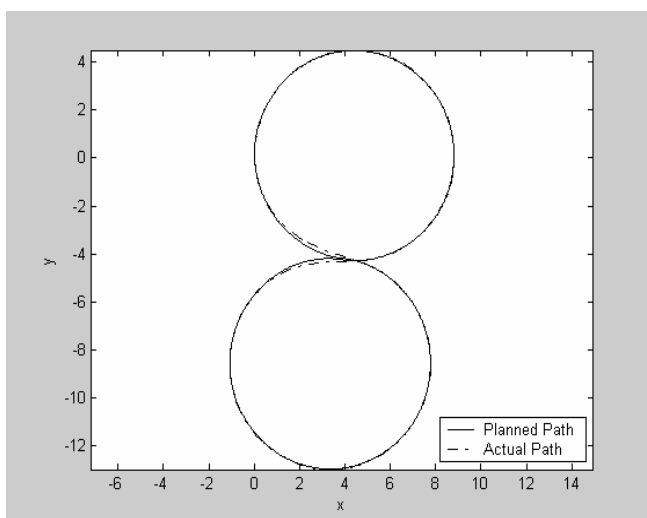
**Fig 82.** Vector-pursuit at 10.4 cm/s with a look-ahead of 2 cm



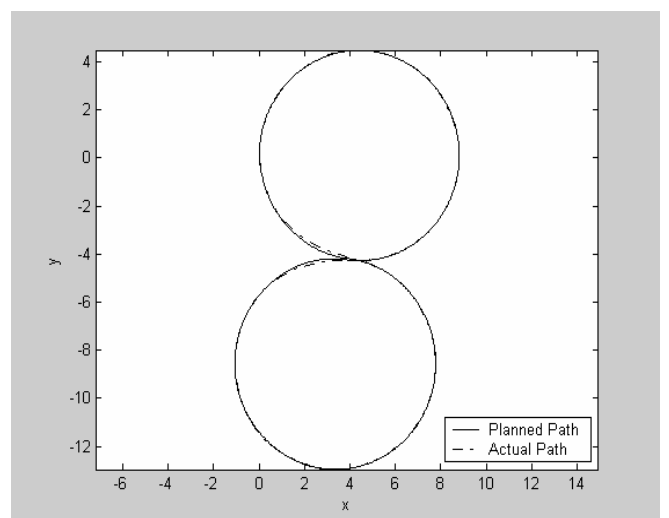
**Fig 83.** Follow-the-carrot at 5.6 cm/s with a look-ahead of 2 cm



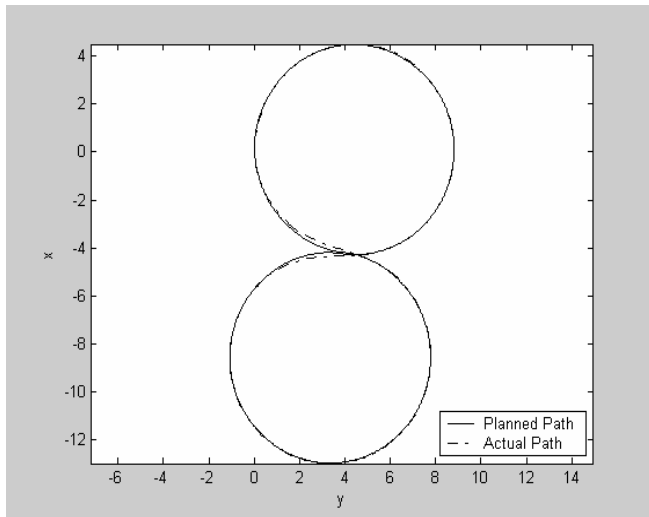
**Fig 84.** Follow-the-carrot at 10.4 cm/s with a look-ahead of 2 cm



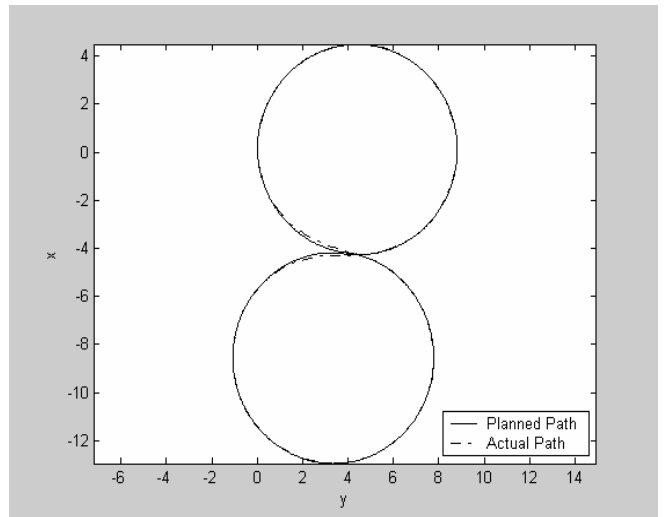
**Fig 85.** Pure-pursuit at 5.6 cm/s with a look-ahead of 2 cm



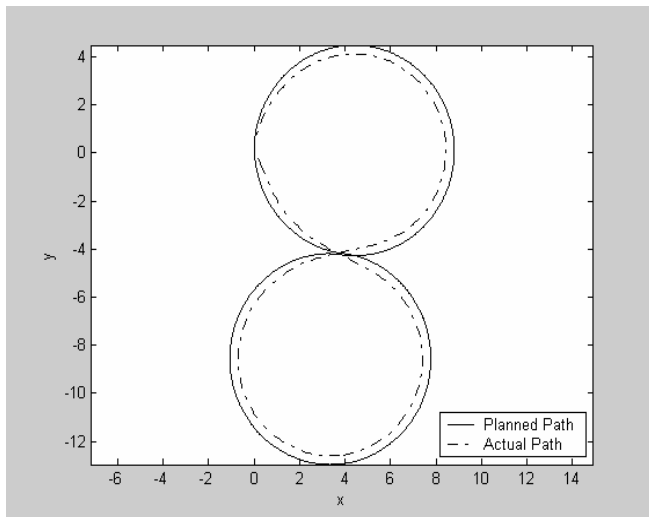
**Fig 86.** Pure-pursuit at 10.4 cm/s with a look-ahead of 2 cm



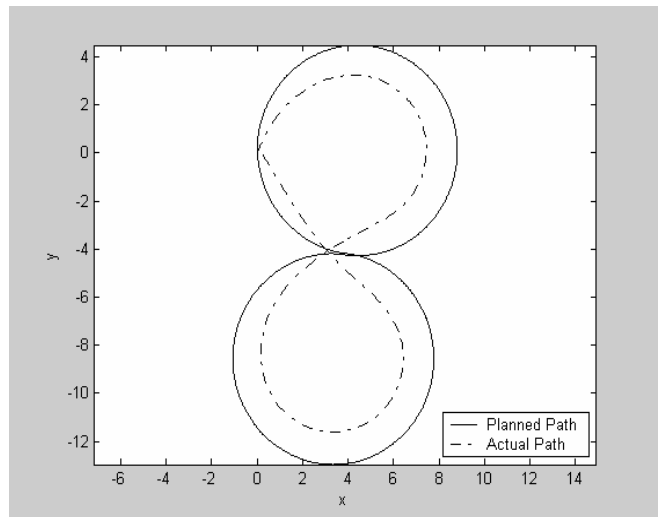
**Fig 87.** Vector-pursuit at 5.6 cm/s with a look-ahead of 2 cm



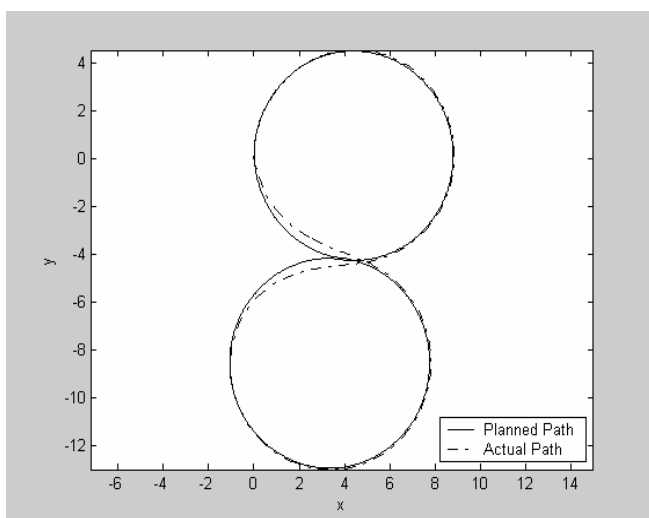
**Fig 88.** Vector-pursuit at 10.4 cm/s with a look-ahead of 2 cm



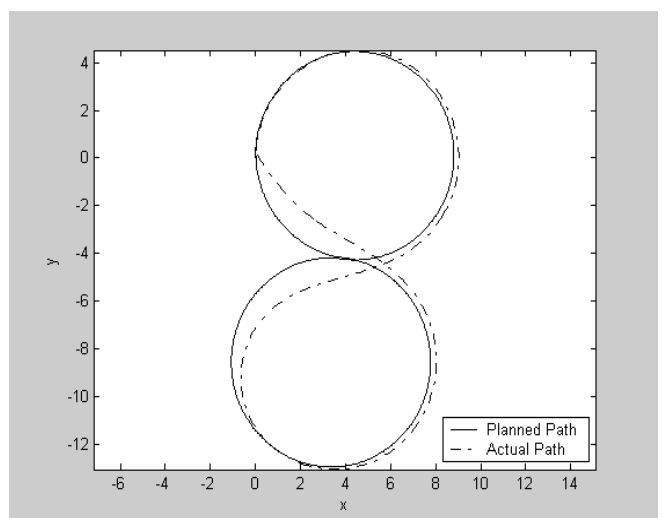
**Fig 89.** Follow-the-carrot at 5.6 cm/s with a look-ahead of 3 cm



**Fig 90.** Follow-the-carrot at 5.6 cm/s with a look-ahead of 5 cm

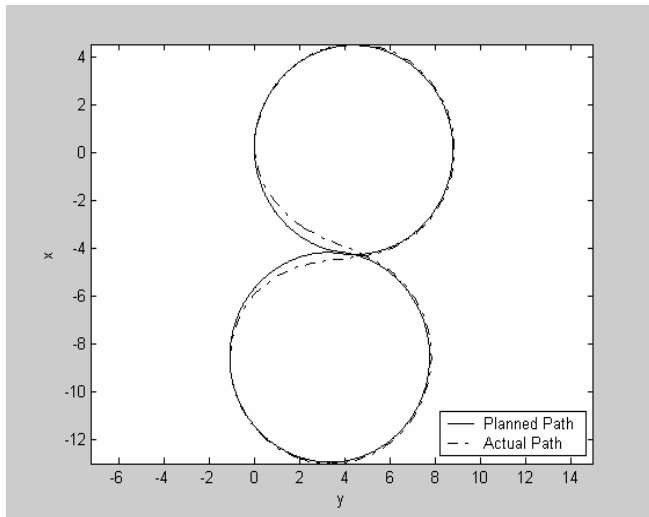


**Fig 91.** Pure-pursuit at 5.6 cm/s with a look-ahead of 3 cm

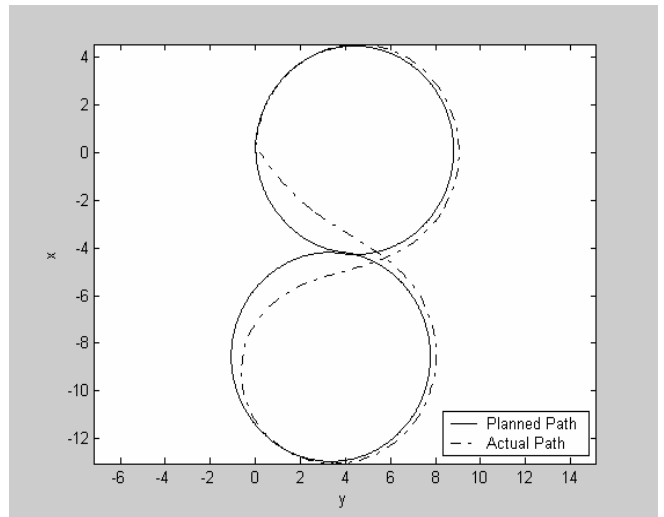


**Fig 92.** Pure-pursuit at 5.6 cm/s with a look-ahead of 5 cm

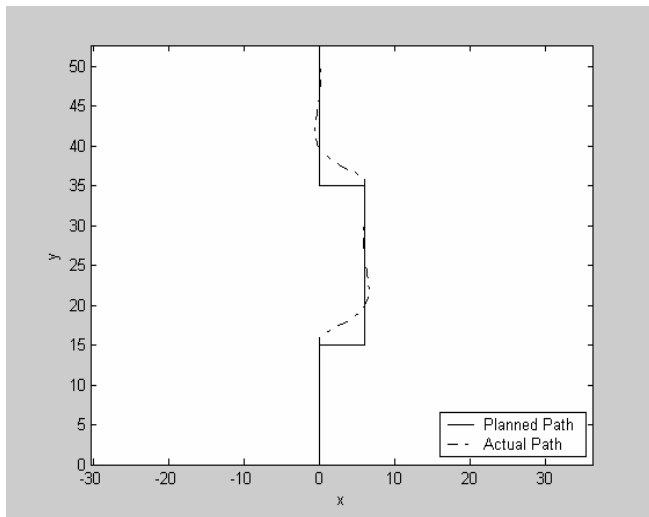




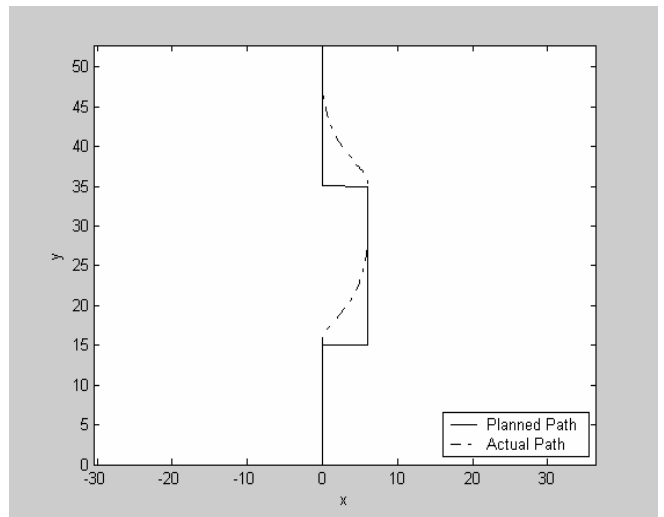
**Fig 93.** Vector-pursuit at 5.6 cm/s with a look-ahead of 3 cm



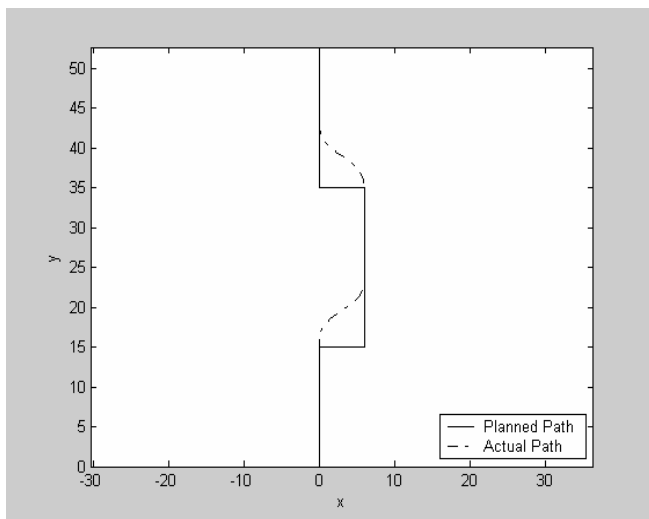
**Fig 94.** Vector-pursuit at 5.6 cm/s with a look-ahead of 5 cm



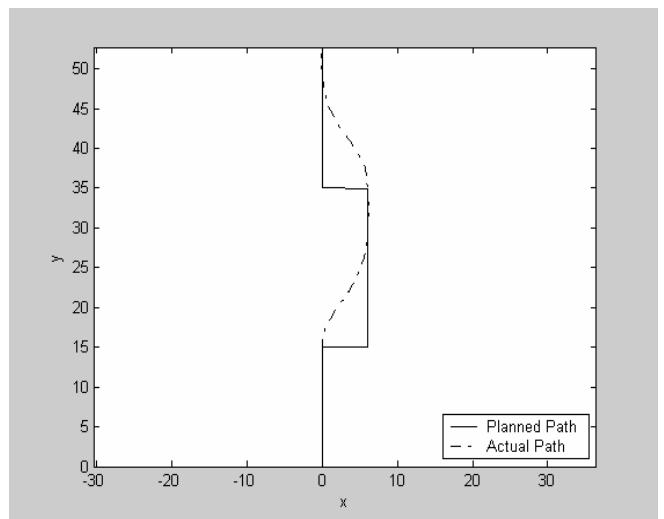
**Fig 95.** Follow-the-carrot at 13.6 cm/s with a look-ahead of 2 cm



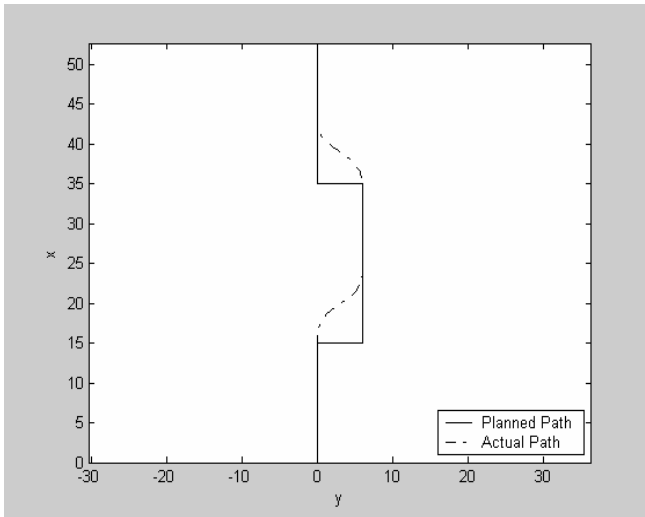
**Fig 96.** Follow-the-carrot at 13.6 cm/s with a look-ahead of 5 cm



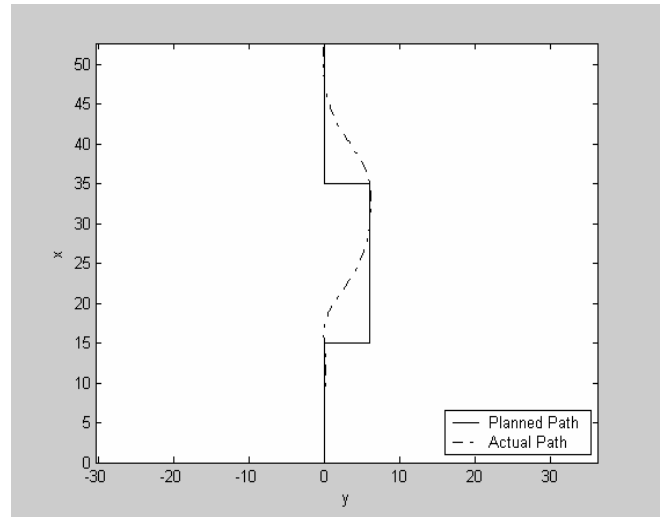
**Fig 97.** Pure-pursuit at 13.6 cm/s with a look-ahead of 2 cm



**Fig 98.** Pure-pursuit at 13.6 cm/s with a look-ahead of 5 cm



**Fig 99.** Vector-pursuit at 13.6 cm/s with a look-ahead of 2 cm



**Fig 100.** Vector-pursuit at 13.6 cm/s with a look-ahead of 5 cm

## 9 References

- [1] Ronald C. Arkin; Behaviour-Based Robotics, The MIT Press, Cambridge, 1998
- [2] G W. Lucas; A Tutorial and Elementary Trajectory Model for the Differential Steering System of Robot Wheel Actuators, The Rossum Project, 2001
- [3] J S. Wit; Vector Pursuit Path Tracking for Autonomous Ground vehicles, Ph.D thesis, University of Florida, 2000
- [4] T. Hellström; Autonomous Navigation for Forest Machines, Department of Computing Science, Umeå University, Umeå Sweden, 2002
- [5] R C. Coulter; Implementation of the Pure Pursuit Path Tracking Algorithm, Robotics Institute, Carnegie Mellon University, January, 1992
- [6] G. Dudek, M. Jenkin; Computational Principles of Mobile Robotics, Cambridge University Press, New York, 2000
- [7] E. Pärt-Enander, P. Isaksson; Användarhandledning för Matlab 4.2, Uppsala Universitet, 1997
- [8] K-team S.A; Khepera User Manual version 5.0, Lausanne, 1998
- [9] Foley, van Dam; Computer Graphics - Principles and Practice, Secod Edition, USA, 1999
- [10] “Unmanned vehicles: University of Florida” at:  
[http://www.me.ufl.edu/~webber/web1/pages/research\\_areas/vehcile\\_control.htm](http://www.me.ufl.edu/~webber/web1/pages/research_areas/vehcile_control.htm)
- [11] “Control laws: PID based control equations” at:  
[http://abrobotics.tripod.com/ControlLaws/PID\\_ControlLaws.htm](http://abrobotics.tripod.com/ControlLaws/PID_ControlLaws.htm)
- [12] “msdn:Microsoft” at <http://msdn.microsoft.com/library/default.asp?url=/>