EXAMINATION

Course: 5DA001/Non-linear optimization
Teacher in charge: Y Löwstedt/N Börlin
Semester: VT-08
Date: 2008-03-25
Time: 09.00–15.00

Name: ________________________________

Personal ID number: ________________________________

Unique code for this examination: 7

Note!
This examination will be graded anonymously. This sheet will be removed before the teacher receives the rest of the examination. The above code must therefore be on all other pages when you submit the examination to the examination supervisory staff. Memorize your code since it will be used as reference when the results are published. Furthermore,

• Start every question on a new sheet of paper.
• Write your code and the question number in the top right corner of every paper.
• Write on one side of each paper only.
• Mark the questions you have solved with a cross on the next page.
• Sort your papers before handing in your questions.
• The solutions should be neatly written. The train of thought should be easy to follow. All non-obvious assumptions must be explicitly stated.
• All relevant calculations must be presented.
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Question 1  (1+2+3 points)

a. Given a function \( f(x) \in \mathbb{R}, x \in \mathbb{R}^n \) and a point \( x_k \), give the definition of a descent direction for \( f \) at \( x_k \).

b. Assume \( f(x) \) is differentiable. How can you use first order information to determine whether a search direction \( p \) is a descent direction?

c. Consider the Newton equation

\[
\nabla^2 f(x_k)p = -\nabla f(x_k).
\]

Under what circumstances are you guaranteed that the solution \( p \) of the Newton equation is a descent direction?

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Question 2  (4 points)

a. Describe the steps in the general Branch and Bound-algorithm.

b. Which of the steps in the general algorithm needs to be specified in order to get an algorithm for a specific problem?

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Question 3  (2+2+5+2 points)

a. Under what circumstances does a locally convergent minimization method converge from a starting approximation \( x_0 \) towards a minimizer \( x^* \)?

b. Under what circumstances does a globally convergent minimization method converge from a starting approximation \( x_0 \) towards a minimizer \( x^* \)?

c. Suggest and describe two globalization techniques that modify a locally convergent method to become globally convergent.

d. What properties are desired from the globalization techniques close and far away from the solution, respectively?

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Question 4  (4 points)

A discrete problem with variables \( x = \{x_1, x_2, \ldots, x_n\} \), and objective function \( f(x) \) and a neighbourhood \( N(x) \) are given.

a. Give a complete description of the simulated annealing algorithm for this problem.

b. What is required to make sure that this algorithm can find the global optimum from any initial point?
Question 5  (5 points)

Consider the constrained problem

\[ \min_x f(x) \quad \text{s.t.} \quad Ax = b. \]

Rewrite the problem into an unconstrained problem and specify gradient and hessian for the new problem.

Question 6  (5 points)

The personnel of a company has to be divided into two offices of the same size. Some people in the first office still need to make phone calls to people in the second office, but the company wants to keep the phone costs down. The company now wants to find a good way to split the personnel into two offices.

a. Give a reasonable mathematical formulation of this optimisation problem. Clearly state which variables you have, what the objective function is and what the constraints are.

b. Which type of algorithm would you use to solve the optimisations problem you have formulated?

Question 7  (5 points)

In order to solve a non-linear minimization problem numerically, several functions need to be derived and implemented, e.g. the objective function \( f(x) \), the gradient function \( g(x) = \nabla f(x) \), and the hessian function \( H(x) = \nabla^2 f(x) \). However, few implementations are error-free.

Assume your optimization method is the modified Newton method with line-search. How would incorrect implementations of \( f(x) \), \( g(x) \), and/or \( H(x) \) affect your optimization method? E.g. would the method still find the correct optimizer? How would the convergence speed be affected? Would the method converge to another point? Or fail to converge?

The question of whether a computer can think is no more interesting than the question of whether a submarine can swim - Edgar W. Dijkstra