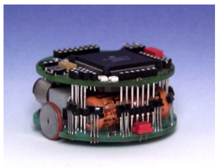
Forward Kinematics for the Khepera Robot



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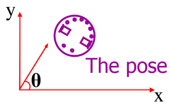
Forward kinematics

Task:

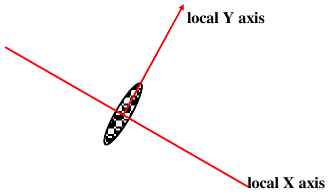
Standing in the pose (x, y, θ) at time t,
Determine the pose (x', y', θ') at time $t + \delta t$
given the control parameters (v_r, v_l) !



The pose (x, y, θ) is defined in a global coordinate system

One rolling wheel

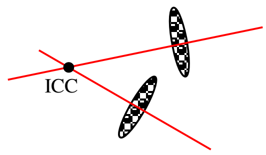
- **Motion along the local Y axis is known as *roll***
- **Everything else is known as *slip***
- **Slip is assumed NOT to occur !**



For one turn of the wheel:
The center moves a distance $2\pi r_w$
where r_w is the radius of the wheel

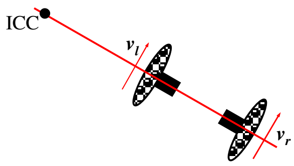
Many rolling wheels

- **Each wheel must roll along its own Y axis**
- **I.e. A center point for the rotation exists! It is called** ICC ***(Instantaneous Center of Curvature)*** **or** ICR ***(Instantaneous Center of Rotation)***
- **The speed of each wheel has to be consistent with a rigid rotation of the vehicle**



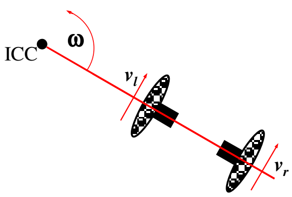
Differential drive

- **Pairs of wheels mounted on a common axis**
- **If the wheels are rotating on the ground: There is a point ICC !**
- **By varying (v_r, v_l), ICC moves and different trajectories are chosen**



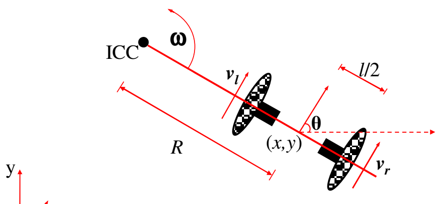
Angular velocity ω

- **Each wheel rotates around ICC along a circle with radius r**
- **The speed $v = 2\pi r / T$ where T is the time it would take to complete one full turn around ICC**
- **The angular velocity ω is $2\pi / T$ (rad / sec)**
- $\omega = 2\pi / T = 2\pi r / rT = v / r \Rightarrow$ $\underline{\omega r = v}$
- **Same ω for both wheels**



Forward kinematics

Standing in the pose (x,y,θ) at time t,
determine the pose (x',y',θ') at time $t + \delta t$
given the control parameters (v_r, v_l) !



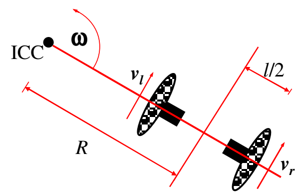
The pose (x,y,θ) is defined in a global coordinate system

Angular velocity ω

For left and right wheel:

1) $\omega\,(R+l/2) = v_r$

2) $\omega\,(R - l/2) = v_l$



Solve for ω and R :

3) $R = l/2(v_l+v_r) / (v_r-v_l)$

4) $\omega = (v_r-v_l) / l$

Forward kinematics

Rotate around ICC with angular velocity ω for δt seconds:

Position at time $t + \delta t$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) \\ \sin(\omega\delta t) & \cos(\omega\delta t) \end{bmatrix} \begin{bmatrix} x-ICC_x \\ y-ICC_y \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \end{bmatrix}$$

Wheel encoders give decoder counts n_r and n_l from time=t to $t + \delta t$.
In general: $n\ step = v\ \delta t \Rightarrow v = n\ step/\delta t$
where *step* is the length (mm) of one decoder tick.
Insert in 3) and 4):

$R = l/2(v_l+v_r) / (v_r-v_l) = l/2(n_l+n_r) / (n_r-n_l)$

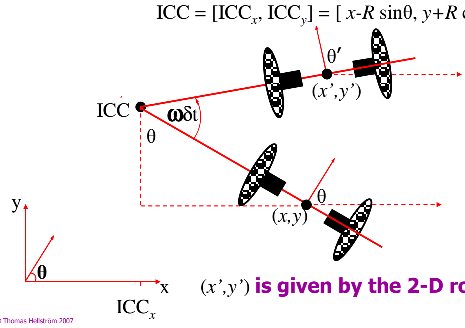
$\omega\delta t = (v_r-v_l)\ \delta t / l = (n_r-n_l) step\ / l$

Forward kinematics

Rotate around ICC with angular velocity ω for δt seconds:

$\theta' = \omega\delta t + \theta.$

$ICC = [ICC_x, ICC_y] = [\ x-R\sin\theta,\ y+R\cos\theta].$



(x',y') **is given by the 2-D rotational matrix:**

Inverse kinematics

Task:

Standing in the pose (x,y,θ) at time t,
Determine control parameters (v_r, v_l) such that
the pose is (x',y',θ') at time $t + \delta t$
Often infinitely many solutions.
Hard to find the optimal solution.

Often easy to find ONE solution by
decomposing the problem and controlling only
a few DOF at a time

Forward kinematics summary

The robot is standing at (x,y,θ) and moves n_l, n_r counts
during one time step.

$R = l/2(n_l+n_r) / (n_r-n_l)$

$\omega\delta t = (n_r-n_l) step\ / l$

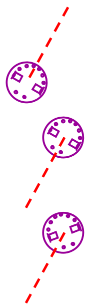
$ICC = [ICC_x, ICC_y] = [\ x-R\sin\theta,\ y+R\cos\theta].$

New pose (x',y',θ') :

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x-ICC_x \\ y-ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

Note: independent of δt

Inverse kinematics
For the Khepera



1. Turn so that the wheels are parallel to the line between the original and final position of the robot origin:

   $v_r = -v_l = v_{rot}$

2. Drive straight until the robot's origin coincides with the destination:

   $v_r = v_l = v_{ahead}$

3. Rotate in order to achieve the desired final orientation:

   $v_r = -v_l = v_{rot}$

Inverse kinematics
For the Khepera

$v_r = v_l \Rightarrow$

$n_r = n_l \Rightarrow R = \infty$, $\omega\delta t = 0$

The robot will move in a straight line. I.e.: θ remains the same

$v_r = -v_l \Rightarrow$

$n_r = -n_l \Rightarrow R = 0$, $\omega\delta t = 2 n_l\, step\ /\ l$

ICC = [ICC_x, ICC_y] = [x, y].

$x' = x$, $y' = y$, $\theta' = \theta + \omega\delta t$

The robot will rotate in place about ICC. *I.e.: any θ is reachable. (x,y) remains the same*

Holonomicity

By definition, a robot is holonomic if it can change all degrees of freedom (all parts of the pose) simultaneously and independently

The "Swedish wheel" can roll in two directions

Omniwheel is holonomic

Nonholonomicity

A robot is non-holonomic if there are constraints in the way its pose can change

Most vehicles are Non-holonomic: