A real-time localization system for compactors

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Abstract

An operator-aiding system for compactors must incorporate a localization system. In this paper, we consider Real-Time Kinematic GPS and we present the solutions we have developed to maintain the positioning error lower than 0.2 m even during the satellite (limited) masking phases. By combining gyro, steering angle and speed measurements, we show that it is possible to obtain satisfying performances in dead reckoning navigation with a low-cost internal sensor set. A forward-backward Kalman filter is proposed to deal with particularly long maskings, which typically occur when the machine works under bridges. Results of experiments carried out with an instrumented machine are presented to validate the proposed solutions.

Key words: Asphalt compaction, operator-aiding system, 2D dynamic localization, Real-Time Kinematic GPS, multi-sensor system, discrete Kalman filter.

1 Introduction

In order to compact the asphalt to the desired density, the compactor performs a prescribed number of passes on each point of the road, with lateral shifts so as to cover the whole width of the road. To achieve this goal, the operator has to memorize not only the number of passes performed, but also where these runs begin and end, and when the vibrations applied to the cylinders have been switched on and off. This work is extremely difficult to perform in the site conditions, and errors induce frequent defects in the final structure. That is why several systems have been imagined to assist the driver in his task. Any such system requires to localize the vehicle with a precision of the order of a decimeter.
In France, the MACC prototype developed by Froumentin and Peyret [3] essentially aimed at defining a man-machine interface for this application. Using only a Real-Time Kinematic GPS receiver to localize the machine, the MACC system displayed the current position of the vehicle, its speed, and, thanks to a colour code, the number of passes accomplished on the rolling zone. The prototype convinced the industrials of the interest of such a system, and also stressed a major issue: the choice of a reliable localization system. Solutions which require a heavy infrastructure (beacons for instance, as the systems described in [1,4]) are not acceptable for compactors, mainly because of their cost. And RTK GPS, although it allows an accuracy of the order of a centimeter, is not satisfying either. Indeed, when used alone (such as in the system presented in [7]), or with an insufficient number of additional sensors (a unique magnetic compass in the case of the AutoPave system described in [13]), GPS is unable to issue precise position data when the line of sight between the mobile receiver and the satellites is blocked by trees or buildings. The problem also occurs when the presence of cuts or machines causes reflections of signals and multipath errors. In such situations, additional sensors are required to evaluate the position of the vehicle using dead-reckoning navigation.

On the basis of the MACC prototype, the Brite-Euram CIRC (Computer Integrated Road Construction) project has defined and developed the operator aiding system for compactors CIRCOM whose main objectives are [9]:

- assisting the operator in his task, so that he can perform the exact number of passes on each point of the rolling zone with a correct speed,
- storing the current compaction state of the road, in order to feed the site data base and control the quality of the achieved work.

To this end, the CIRCOM prototype presents the following improvements:

- a more advanced man-machine interface, which displays the road compaction state in a more realistic and natural way,
- a localization system which fuses by Kalman filtering GPS positions and the measures of internal sensors, suitable for maintaining a position estimate when a satellite masking situation arises. The required positioning precision in the horizontal plane is ±20 cm.

Tests carried out with an instrumented compactor have already shown that the CIRCOM product gave good results, even during masking situations [8]. But the sensor chosen to measure the vehicle heading variations, a fibre-optic gyro, is expensive and significantly increases the price of the localization system.

As a consequence, our first concern was to find a low-cost internal sensor set to cope with satellite maskings: the worst case we have to take into account is a 2 × 2 lane bridge which crosses the work site with a 45° angle, which corresponds to 35 m to be compacted without GPS updating. Added to the
distance travelled during the on-the-fly re-initialization of the mobile receiver, a section as long as 100 m may be travelled in dead-reckoning. In a typical situation such as passing under a bridge, the system should be accurate enough to maintain the positioning error under 20 cm, as defined by the end-users. Our solution yields satisfying real-time performances for short term dead-reckoning navigation periods, but it does not suffice when compacting under bridges is considered. We propose a solution which uses a smoother based on a forward-backward Kalman filter. Shortly after GPS measures are again available, the algorithm re-estimates the trajectory followed by the compactor, and hence greatly improves the localization precision.

The article is organized as follows. In section 2, the Kalman filter equations are detailed, together with the compactor and GPS measurement modelling. Section 3 explains the choices we made to equip the machine with the internal sensors required for dead-reckoning navigation. In section 4, the experimental protocol and the obtained results are presented to validate our solution. Finally, the Kalman smoother is detailed in section 5 and conclusions are presented in section 6.

2 Real-time localization by Kalman filtering

In the field of mobile robots localization, the main interest of the Kalman filter stems from its ability to estimate the vehicle position from a number of measurements which are [5]:

- incomplete: linked to some but not all of the variables of interest
- indirect: related indirectly to the quantities of interest
- intermittent: available at irregularly spaced time instants
- inexact: corrupted by many forms of errors

To apply the Kalman filter, the vehicle is modelled as a dynamic system which is excited by noise and whose sensor measures are corrupted by noise. Through the knowledge of the statistics of these noises, it is possible to calculate an estimate of the system state even if the sensors yield inexact measures. To achieve this computation, we need to model not only the vehicle and its evolution (which give the prediction equations necessary for dead-reckoning navigation between two GPS measures), but also the GPS position data, from which we deduce the observation equations used during the updating step of the Kalman filter.
2.1 Vehicle modelling and prediction equations

The compactor is an articulated vehicle: the rear body, denoted \( S_2 \), can rotate relative to the front body, denoted \( S_1 \), thanks to the joint located in \( C \) (see figure 1). Assuming that its motions are restricted to an horizontal plane, the man-machine interface needs the \((x,y)\) co-ordinates of the front roll centre \( O_1 \) and the front body heading \( \psi \) in the work site frame \( R_0(O,\overrightarrow{x_0},\overrightarrow{y_0},\overrightarrow{z_0}) \). These three parameters define the state vector of the studied system. Thus, at discrete time instant \( t_i \), we have:

\[
\begin{bmatrix}
    x(i) \\
    y(i) \\
    \psi(i)
\end{bmatrix} = \begin{bmatrix}
    x(i) \\
    y(i) \\
    \psi(i)
\end{bmatrix} - \begin{bmatrix}
    x(i-1) \\
    y(i-1) \\
    \psi(i-1)
\end{bmatrix}
\]

Assuming that the front roll does not slip laterally, and denoting \( \delta \) the elementary translation along axis \( \overrightarrow{z_0} \), and \( \theta \) the elementary rotation around axis \( \overrightarrow{z_0} \), we obtain the following kinematic model,

\[
\begin{align*}
    x(i) &= x(i-1) + \delta(u(i)) \cos(\psi(i-1)) \\
    y(i) &= y(i-1) + \delta(u(i)) \sin(\psi(i-1)) \\
    \psi(i) &= \psi(i-1) + \theta(u(i))
\end{align*}
\]

where \( u(i) \) is the vector of the internal sensor measures at discrete time instant \( t_i \).

System (2) can be written in vector form as the following prediction equation:

\[
\begin{bmatrix}
    x(i)
\end{bmatrix} = \begin{bmatrix}
    f(x(i-1), u(i))
\end{bmatrix}
\]
2.2 GPS measurements modelling and observation equations

Since GPS position data are always available to the user with a delay $T$ called GPS latency [6], which must be taken into account. The latency depends on the number of satellites visible by the reference station at time instant $t_j$. Denoting $T_s$ the sampling period, and assuming that the delay is a multiple of $T_s$, the latency $T$ can be written as:

$$T(j) = T_s \times n(j)$$ (4)

Since the mobile GPS antenna is located above the front roll centre, the position given by the GPS at observation instant $t_j$ is simply the position of the vehicle at time instant $t_j - T$:

$$\begin{align*}
  x_{GPS}(j) &= x(j - n(j)) \\
  y_{GPS}(j) &= y(j - n(j))
\end{align*}$$ (5)

Remark 1 System (5) implies that GPS measurements are considered to be synchronous with a sampling instant. The sampling period will have to be sufficiently small for this assumption to be valid. This will also make equation (4) a good approximation of the latency.

Considering the distance travelled by the vehicle between time instants $t_j - T$ and $t_j$, and using the kinematic model given by equation (2), system (5) becomes:

$$\begin{align*}
  x_{GPS}(j) &= x(j) - \sum_{k=j-n(j)+1}^{j} \delta(u(k)) \cos\left(\psi(j) - \sum_{l=k}^{j} \theta(u(l))\right) \\
  y_{GPS}(j) &= y(j) - \sum_{k=j-n(j)+1}^{j} \delta(u(k)) \sin\left(\psi(j) - \sum_{l=k}^{j} \theta(u(l))\right)
\end{align*}$$ (6)

which can be summarized by the following observation equation:

$$z(j) = g(x(j - 1), u^a(j))$$ (7)

where $u^a(j)$ is the vector of the proprioceptive measures performed between instants $t_j - (n(j) - 1) \times T_s$ and $t_j$, and $z$ is the 2-dimensional vector of the system outputs.

Remark 2 The new time index $j$, instead of $i$ in equation (3), is meant as a reminder of the fact that GPS measurements are not available at each sampling instant. The typical frequency for equation (7) is 1 Hz, as opposed to 100 Hz for equation (3) in our system.
Remark 3 If the GPS antenna is not located above $O_1$, the problem is not really different, as shown in [8]. But this choice simplifies the Kalman filter equations, and facilitates later comparison between the dead reckoning navigation results and the GPS positioning data.

2.3 Extended Kalman filter equations

Both equations (3) and (7) define a discrete-time nonlinear system where the state vector is given by the front roll 2D posture, the inputs, by vector $u^a$, and the outputs, by the GPS measures $z$. Actually, these equations are corrupted by noises which are supposed to be white, gaussian, uncorrelated with each other, zero-mean, and of known variances. The state-space description becomes:

\[
\begin{cases}
  x(i) = f(x(i-1), u^a(i)) + \alpha(i) \\
  u^{a*}(i) = u^a(i) + \beta(i) \\
  z(j) = g(x(j), u^a(j)) + \gamma(j)
\end{cases}
\]

(8)

where

- $\alpha$ (covariance matrix: $Q_{\alpha}$) is the model noise, which represents the effect of slippage or skid of the roll on the ground, plus the effects of errors on the robot parameters.
- $u^{a*}$ is the measure of the deterministic input $u^a$.
- $\beta$ (covariance matrix: $Q_{\beta}$) is a white noise which affects $u^{a*}$.
- $\gamma$ (covariance matrix: $Q_{\gamma}$) is the noise which corrupts the exteroceptive data. It is well known that, in the case of the GPS, $\gamma$ does not satisfy some of the previously formulated hypotheses, since the observations $x_{GPS}$ and $y_{GPS}$ are affected by a low-frequency error. We have chosen to ignore it, since the required precision of our localization system is about 20 cm and the amplitude of the perturbation is usually smaller than three centimeters.

The tuning of the covariance matrices of these noises is detailed in appendix A.

The system being nonlinear, we use an Extended Kalman Filter (EKF), which requires to linearize the state-space description (8) around the current estimated state. The resulting localization algorithm works in two steps: one prediction step calculated at each sampling period $T_s$, and an additional updating step, which occurs only when a GPS measure is available.

One denotes $\hat{x}(k|l)$ the state estimate at time instant $t_k$ with all the GPS measures known until instant $t_l$, and $P(k|l)$ the covariance matrix of the estimate.
Based on the knowledge of the measured input \( u^a(i) \), the predicted state and the estimate error covariance are computed as follows:

\[
\begin{align*}
\dot{x}(i| i-1) &= f(\dot{x}(i-1|i-1), u^a(i)) \\
\text{P}(i| i-1) &= \text{A}(i)\text{P}(i-1|i-1)\text{A}(i)^T + \text{B}(i)\text{Q}_\beta \text{B}(i)^T + \text{Q}_\alpha
\end{align*}
\]

with the following Jacobian matrices:

\[
\begin{align*}
\text{A}(i) &= \frac{\partial f(x, u^a)}{\partial x} \bigg|_{\dot{x}(i-1|i-1), u^a(i)} \\
\text{B}(i) &= \frac{\partial f(x, u^a)}{\partial u^a} \bigg|_{\dot{x}(i-1|i-1), u^a(i)}
\end{align*}
\]

At observation instant \( t_j \), the predicted state \( \dot{x}(j| j-1) \) and its associated covariance matrix are corrected as follows:

\[
\begin{align*}
\dot{x}(j| j) &= \dot{x}(j| j-1) + \text{K}(j) (z(j) - g(\dot{x}(j| j-1), u^a(j))) \\
\text{P}(j| j) &= \text{P}(j| j-1) - \text{K}(j) (\text{C}(j)\text{P}(j| j-1) + \text{S}(j)^T)
\end{align*}
\]

where, from the correlation term:

\[
\text{S}(j) = \text{B}(j)\text{Q}_\beta \text{D}(j)^T
\]

and the Jacobian matrices:

\[
\begin{align*}
\text{B}(j) &= \frac{\partial f(x, u^a)}{\partial u^a} \bigg|_{\dot{x}(j| j-1), u^a(j)} \\
\text{C}(j) &= \frac{\partial g(x, u^a)}{\partial x} \bigg|_{\dot{x}(j| j-1), u^a(j)} \\
\text{D}(j) &= \frac{\partial g(x, u^a)}{\partial u^a} \bigg|_{\dot{x}(j| j-1), u^a(j)}
\end{align*}
\]

the Kalman gain \( \text{K}(j) \) is calculated as follows:

\[
\text{K}(j) = (\text{P}(j| j-1)\text{C}(j)^T + \text{S}(j)) (\text{C}(j)\text{P}(j| j-1)\text{C}(j)^T + \text{D}(j)\text{Q}_\beta \text{D}(j)^T + \text{Q}_\gamma + \text{C}(j)\text{S}(j) + \text{S}(j)^T\text{C}(j)^T)^{-1}
\]
The Kalman filter is initialized after 2 m of a roughly straight-line motion, which allows to estimate \( \hat{x}(0|0) \), \( \hat{y}(0|0) \) and \( \hat{\psi}(0|0) \) from the sole GPS position data.

3 Internal sensors choice and resulting equations

Contrary to [12], the use of an inertial navigation system has not been considered here, mainly because of its cost. Moreover, powerful vertical vibrations are applied to the machine cylinders for compacting, and we ignore to what extent they can corrupt the accelerometer measurements.

Another solution would be odometry, but it cannot be used in our application. Indeed, this would imply mounting special-purpose additional wheels on the vehicle. These wheels should operate on hot, sticky asphalt, in the presence of lateral slip during manoeuvres. Succeeding to design a low-cost, easy-to-mount, reliable system is doubtful.

As a consequence, the sensor configuration consists of four internal sensors, namely an encoder attached to the front cylinder, a Doppler radar, a vibrating gyro fixed on the driver’s cabin, and a system composed of two sensors to measure the lengthening of the steering jacks, which allows to calculate the steering angle \( \xi \) (see figure 1).

3.1 The measurement of the elementary translation

Two sensors are necessary to measure the machine velocity. The encoder gives the direction of the movement (forward or backward) and an estimation of the elementary translation at low speed. Indeed, the radar does not work under \( V_{min} = 0.4 \text{ m/s} \), situation which happens every time the compactor stops and inverts its speed, or during manœuvre phases.

Above this minimum limit, we use radar information, because when the compactor is back to its normal speed (about 1 m/s), the vibrations applied to the cylinders make them slip and even take off, resulting in useless encoder data.

Concretely, at each sampling period, the speed given by the cylinder encoder is compared to a threshold set at 0.4 m/s. If the measured speed is lower than this value, the encoder measure is used to compute the elementary translation \( \delta(i) \). If not, after a delay of 2 s which avoids excessive sensor commutations, the radar is used if the speed issued by the encoder is still greater than the
threshold value. This precaution avoids that the elementary translation measurement be biased. Indeed, switching between the two measured speeds is a nonlinear operation and the resulting output is biased if the vehicle speed oscillates around the threshold.

3.2 The measurement of the elementary rotation

An obvious way to obtain the rotation speed of the machine front body is to use a gyro, as in [8]. Dead reckoning navigation results are very satisfying, provided the sensor offset does not drift. This is only achievable with a reliable but expensive instrument (a fibre-optic gyro for instance). Instead, we prefer using a low-cost vibrating gyro combined with another measure of the elementary rotation \( \theta \). Indeed, by measuring the compactor steering angle and its variations, we show that it is possible to evaluate \( \theta \).

Let us call \( \vec{V}_{S,R}(P) \) the speed of a point \( P \) belonging to the body \((S)\) in its movement relative to the reference frame \((R)\), and \( \vec{\omega}_{S,R} \) the rotation speed of the body \((S)\) relative to the frame \((R)\). The speeds of the cylinder centres \( O_1 \) and \( O_2 \) relative to the work site frame \( R_0 \) can be written as follows:

\[
\begin{align*}
\vec{V}_{S_1,R_0}(O_1) &= V_{1t}\vec{x}_1 + V_{1n}\vec{y}_1 \\
\vec{V}_{S_2,R_0}(O_2) &= V_{2t}\vec{x}_2 + V_{2n}\vec{y}_2
\end{align*}
\] (18)

Let \( \xi \) be the steering angle between the front and the rear body. Let \( L_1 \) (resp. \( L_2 \)) be the distance between the articulation joint \( C \) and \( O_1 \) (resp. \( O_2 \)). The rotation speeds of the compactor front and rear bodies are given by:

\[
\begin{align*}
\vec{\omega}_{S_1,R_0} &= \dot{\psi}\vec{z}_0 \\
\vec{\omega}_{S_2,R_0} &= (\dot{\psi} + \dot{\xi})\vec{z}_0
\end{align*}
\] (19)

From the following expression,

\[
\vec{V}_{S_2,R_0}(O_2) = \vec{V}_{S_1,R_0}(O_1) + \vec{\omega}_{S_1,R_0} \wedge \overrightarrow{O_1C} + \vec{\omega}_{S_2,R_0} \wedge \overrightarrow{CO_2}
\] (20)

and assuming that the rolls do not slip laterally (i.e. \( V_{1n} = 0 \) and \( V_{2n} = 0 \), we obtain:

\[
\dot{\psi} = -\frac{L_2\dot{\xi} + V_{1t}\sin(\xi)}{L_1\cos(\xi) + L_2}
\] (21)

This equation permits us to replace the gyro by a speed measurement and a steering angle measurement. This last quantity is obtained thanks to two
The steering angle is calculated from the following equation:

$$\xi = \arctan \left( \frac{(l_r^2 - l_l^2) (2(l_r^2 + l_l^2) - (d_1^2 + d_2^2) - 4(l_r^2 + l_l^2))}{(d_2 l_1 + d_1 l_2)(d_1^2 d_2^2 - 4l_r^2 l_l^2)} \right)$$

(22)

where \( l_l = \text{dist}(P_{1l}, P_{2l}) \) and \( l_r = \text{dist}(P_{1r}, P_{2r}) \) are the lengthening sensor measures.

Thus, we have two possibilities to evaluate the front body rotation speed: either using the gyro, or using the steering angle measurement. We use both of them through the following method. Most of the time, we use the front body rotation speed reconstructed by equation (21), and we substitute gyro data only during the short intervals when gyro and reconstructed rotation speeds strongly disagree, which typically occurs during manœuvre phases. Indeed, in these conditions, cylinder lateral slippage occurs and the non-skidding assumption of equation (21) is no longer satisfied, which makes the reconstructed speed unreliable.

As a consequence, the elementary rotation \( \theta \) is most of the time computed thanks to the cylinder encoder or the radar, and the steering angle measurements. In addition, since gyros are often subject to drift, the gyro bias \( b \) is updated by the Kalman filter, based on the following model:

$$\begin{cases} 
    b(i) = b(i - 1) + \alpha_b(i) & \text{(prediction)} \\
    \omega_{\text{gyro}}(i) - \dot{\psi}_r(i) = b(i) + \gamma_b(i) & \text{(observation)}
\end{cases}$$

(23)

where \( \alpha_b \) and \( \gamma_b \) are uncorrelated, zero-mean, gaussian white noises.

When the difference between the corrected gyro speed and the value given by
equation (21) exceeds a predefined limit, we use the gyro to estimate the elementary rotation $\theta$. In this case, the bias estimation is obviously not updated.

3.3 Resulting equations

To summarize, the input vector of the studied system is given at instant $t_i$ by:

$$u(i) = [\Delta q_{cyl}(i), \delta_{radar}(i), l_l(i), l_r(i) \Delta l_l(i), \Delta l_r(i), \omega_{gyro}(i)]^T$$  \hspace{1cm} (24)

where

- $\Delta q_{cyl}$ is the elementary rotation of the front cylinder measured by the encoder,
- $\delta_{radar}$ is the elementary translation of the front body measured by the radar,
- $\Delta l_l$ and $\Delta l_r$ are the elementary variations of the steering jack lengths,
- $\omega_{gyro}$ is the rotation speed issued by the gyrometer.

At low speed, denoting $R_{cyl}$ the front cylinder radius and $\hat{b}(i)$ the current estimation of the gyro bias, the elementary displacements are given either by:

$$\begin{cases}
\delta(i) = R_{cyl} \Delta q_{cyl}(i) \\
\theta(i) = T_s(\omega_{gyro}(i) - \hat{b}(i))
\end{cases}$$  \hspace{1cm} (25)

when encoder and gyro are used, or by:

$$\begin{cases}
\delta(i) = R_{cyl} \Delta q_{cyl}(i) \\
\theta(i) = \frac{-L_2 \Delta \xi(i) + R_{cyl} \sin(\xi(i)) \Delta q_{cyl}(i)}{L_1 \cos(\xi(i)) + L_2}
\end{cases}$$  \hspace{1cm} (26)

when encoder and steering angle are used.

And at higher speed, denoting $D_{radar}$, the algebraic distance from the front roll center to the radar along axis $\vec{y}_1$ (see figure 2), we have either:

$$\begin{cases}
\delta(i) = \delta_{radar}(i) + D_{radar} T_s(\omega_{gyro}(i) - \hat{b}(i)) \\
\theta(i) = T_s(\omega_{gyro}(i) - \hat{b}(i))
\end{cases}$$  \hspace{1cm} (27)

when radar and gyro are used, or:

$$\begin{cases}
\delta(i) = \frac{-D_{radar} L_2 \Delta \xi(i) + (L_1 \cos(\xi(i)) + L_2) \delta_{radar}(i)}{L_1 \cos(\xi(i)) + L_2 + D_{radar} \sin(\xi(i))} \\
\theta(i) = \frac{-L_2 \Delta \xi(i) + \sin(\xi(i)) \delta_{radar}(i)}{L_1 \cos(\xi(i)) + L_2 + D_{radar} \sin(\xi(i))}
\end{cases}$$  \hspace{1cm} (28)

when radar and steering angle are used.
4 Experimental validation

4.1 Compactor instrumentation

Tests have been carried out on an Albaret VA12DV tandem roller. Exteroceptive data are issued by the RTK GPS receiver TRIMBLE 7400 MSi and proprioceptive measures are given by the following internal sensors:

- one 4×2048 ppr encoder fixed on the front cylinder,
- a DICKEY-JOHN Doppler radar fixed on the front body,
- a VSG2000 vibrating gyro from British Aerospace fixed on the driver’s cabin,
- two magnetostrictive lengthening sensors from MTS.

4.2 Experimental protocol and results

We have recorded, with a sampling period of $T_s = 0.01$ s, all sensor data for different kinds of trajectories, the speed of the compactor varying between 2 and 6 km/h. Tests can be classified into four categories:

1. straight-line motions: 37 tests,
2. circular trajectories: 4 tests,
3. slalom motions: 21 tests,
4. mixed motions, which generally combine straight lines and curves: 15
We have then post-processed the data, simulating GPS maskings in order to evaluate the internal sensor set behaviour in sheer dead reckoning navigation. Localization errors are calculated by comparing the GPS measure to the estimated position at the same instant. For each test, the maximum error is divided by the distance travelled during the masking phase, so as to obtain a relative error. The average relative errors are calculated for each test category, and are shown on figure 4. The performances of the configurations which only use the vibrating gyro, or the steering measurement system, are also presented.

Given these results, it is clear that our solution efficiently corrects problems met when the gyro and steering angle measurements are used separately. On circular trajectories (category 2), assumptions which lead to equation (21) are no longer satisfied, and the reconstructed rotation speed is overestimated. On the other hand, long tests of categories 2 and 4 are critical when the gyro is used alone, because the bias drift cannot be compensated online. By combining the gyro and steering data, we limit estimation errors in dead reckoning navigation. Indeed, with the chosen configuration, the average relative error calculated from all the simulated GPS maskings is 2%. This value allows short term relative navigation phases, but is not low enough when compacting under bridges is considered. In the following section, we propose a method to improve dead reckoning estimation when long masking phases occur.

Fig. 4. Average relative errors for each test category
5 Smoother filter: a solution to deal with long GPS masking phases

5.1 Principle and equations

This technique has been proposed in [11] but has only been simulated. Let \( t_0 \) be the time instant when masking begins, and \( t_N \) the instant when GPS data are again available. The technique consists in storing all the proprioceptive measures, together with the state and covariance estimations, between observation instants \( t_0 \) and \( t_N \) so as to apply a backward Kalman filter from the updated state estimation \( \hat{x}(N|N) \) and the covariance matrix \( P(N|N) \). Since there is no observation (i.e. no GPS position data in our case) between these two instants, only the prediction step occurs and the backward estimation is calculated from:

\[
\begin{align*}
    x_b(i) &= x_b(i + 1) - \delta(u(i + 1)) \cos(\psi_b(i + 1)) \\
    y_b(i) &= y_b(i + 1) - \delta(u(i + 1)) \sin(\psi_b(i + 1)) \\
    \psi_b(i) &= \psi_b(i + 1) - \theta(u(i + 1))
\end{align*}
\] (29)

which can be summarized by the following backward prediction equation:

\[
x_b(i) = f_b(x(i + 1), u(i + 1))
\] (30)

The covariance matrix associated to the backward estimation \( \hat{x}_b(i) \) is computed as follows:

\[
P_b(i) = A_b(i + 1)P_b(i + 1)A_b(i)^T + B_b(i + 1)Q_xB_b(i + 1)^T + Q_o
\] (31)

The final (smoothed) estimation is obtained by weighing the forward and backward predictions by their covariance matrices, according to the following equations:

\[
\begin{align*}
    P(i|N) &= (P(i)^{-1} + P_b(i)^{-1})^{-1} \\
    \hat{x}(i|N) &= P(i|N)(P(i)^{-1}\hat{x}(i) + P_b(i)^{-1}\hat{x}_b(i))
\end{align*}
\] (32)

5.2 Experimental validation

Simulating the same maskings as the ones in section 4.2, we now apply the forward-backward filter so as to see to what extent the relative error in dead reckoning navigation is reduced. Figure 5 shows, for a straight-line motion, the localization errors computed from the backward, forward and smoothed
estimations. The smoothed estimation is a real improvement, since the value of the average relative error calculated from all tests is 0.75%. For straight line motions, the average relative error is as low as 0.6% (about 0.3% in the case of figure 5).

5.3 Practical use

In practice, when compacting under a bridge, a low-precision alert signal may be switched on, based on a threshold on the variance. In such a case, the driver will avoid changing direction under the bridge. He will instead prolong his trajectory (without vibrations if he goes back to a finished area) until he gets GPS data again. Once a GPS position with centimeter-accuracy is available at instant \( t_N \), the backward filter is applied from the updated estimation. A smoothed trajectory of the vehicle is then computed, and the display is updated; hence, the operator has a better idea of the pass he has just performed. He can then proceed to the next run under the bridge.

Of course, this solution does not allow a real-time precise positioning, but still, it remains valuable. Indeed, the main objective of the localization system is to assist the driver in his task, but the second one is to record the precise trajectories and the number of passes achieved by the compactor, in order to control the global quality of the work site: the smoother filter can only enhance this survey.
6 Conclusions

Within the framework of the CIRC european project, it has already been shown that, by using a Kalman estimator which fuses cylinder encoder, radar, fibre-optic gyro and RTK GPS data, satisfying results were obtained for compactor localization. But the price of the chosen gyro is a major drawback when industrialization is considered.

The method we described to combine gyro and steering data allows good performances, since the obtained relative error is 2%, a result which should be compared to the value of 1% generally obtained with odometry for indoor mobile robots [2]. Yet, these performances are not sufficient for compacting under bridges, especially if the on-the-fly re-initialization of the GPS receiver after the satellite masking lasts several tens of seconds (prolonging of several tens of metres the distance travelled in dead-reckoning navigation). We propose to use a smoother to re-evaluate the position estimation made with the internal sensors. As we have seen, the fact that these corrections are only available after the end of each pass is a minor drawback, compared to the precision improvement that this approach permits.

Our solution yields a 2D dynamic localization system which provides the operator with reliable and precise data. The accuracy requirement is not yet satisfied for particularly long masking phases (since the distance travelled without GPS updating can reach 100 metres and the localization error is 0.6%). But several important developments have already shortened the time needed to re-initialize RTK GPS [10]: in a near future, a 20 cm accuracy positioning should be possible with our system whatever the compacting conditions.

A Appendix: Kalman filter tuning

The use of the Kalman filter requires to evaluate properly the different noises which corrupt the observed system. We propose to detail these tunings in the sequel.

A.1 Observation noise tuning

The observation noise $\gamma$ should take into account not only the GPS position inaccuracy ($\pm 3$ cm) but also the error induced by the fact that we assume that GPS measures are synchronous with the sampling period $T_s$. Indeed, an exteroceptive measure valid between time instants $t_{i-1}$ and $t_i$ is used at instant
The resulting error is bounded by the distance travelled during a sampling period. Since the maximum working speed is \( v = 1.5 \text{ m/s} \), this error is lower than 1.5 cm.

As a consequence, the total error is bounded by 4.5 cm. Considering that, for a gaussian noise, 99\% of the error signal is bounded by the ±3σ limit, we have chosen for \( \gamma \) the following standard deviation: \( \sigma = 1.5 \text{ cm} \). The resulting covariance matrix is given by:

\[
Q_\gamma = \begin{bmatrix}
(0.015)^2 & 0 \\
0 & (0.015)^2
\end{bmatrix}
\]

### A.2 Input noise tuning

We recall that the input vector contains the encoder and radar measures, which allow to compute the elementary translations, and the gyro and lengthening sensor measures, from which the elementary rotations are deduced.

For the noise corrupting radar and encoder outputs, we consider that it is given by a ± 1 pulse error by sampling period. Assuming that its distribution is uniform, we obtain:

\[
\sigma^2 = \frac{(\text{linear travel by pulse})^2}{12}
\]

which gives for each sensor:

- \( \text{Var}(\beta_{\Delta_{\text{quad}}}^c) = 2.3 \times 10^{-9} \text{ rad}^2 \)
- \( \text{Var}(\beta_{\delta_{\text{radar}}}^c) = 8.8 \times 10^{-6} \text{ m}^2 \)

The reader may notice that we have not taken into account the slippage of the roll on the ground or the transverse sensitivity of the radar. These phenomena are considered in the model noise tuning.

The tuning of the gyro and lengthening sensor noises is much more delicate, since it directly affects the heading estimation, an essential parameter when dead reckoning navigation is considered. If the variance is too large, the estimate of \( \psi \) will tend to oscillate due to GPS position errors. In this case, the filter may enter masking periods with fairly large heading errors (especially at low speed). On the other hand, if the variance is too small, position data cannot sufficiently update the dead reckoning estimate to prevent it from drifting. The problem is that we do not have any external sensor suitable for a precise measurement of the \( \psi \) angle.
As a consequence, we propose an indirect method to tune the noise of the gyro and the lengthening sensors. Our solution consists in comparing dead reckoning navigation errors for simulated maskings starting at successive time instants. If the resulting estimates differ drastically from each other, it means that the trajectory estimates are pretty much random, depending on the masking start instant. In such a case, the system is clearly useless. By reducing the values of the input noise variances $\text{Var}(\beta_\omega\text{gyro})$ and $\text{Var}(\beta_l)$, it is possible to lower the variance of the heading estimate error. But we must also pay attention to the fact that the position estimate error has to be bounded by the $\pm 3\sigma$ limit. This criterion allows us to experimentally determine the lower bound on the noise variances.

Figure A.1 shows an example of tuning for the lengthening sensor noise variance. Finally, the following results are obtained:

- $\text{Var}(\beta_\omega\text{gyro}) = 5 \times 10^{-4} \text{ (rad/s)}^2$
- $\text{Var}(\beta_l) = 5 \times 10^{-8} \text{ m}^2$

The reader may wonder why the noise is not directly deduced from the technical data of the sensors. The first reason is that the noise observed is higher because part of it is due to the transmission of mechanical vibrations to the
device. The second reason is due to a design trend. The effects of the speed rotation noise and model noise cumulate in equation 9 (term $B(i)Q_\beta B(i)^T + Q_\alpha$); thus $Q_\beta$ can be used to capture some of the effects of model errors, usually represented by $Q_\alpha$, as shown below. The interest of using $Q_\beta$ is that $B(i)$ in $B(i)Q_\beta B(i)^T$ depends on the current heading of the vehicle. Hence, our solution reflects the well-known fact that errors tend to grow faster in the direction orthogonal to the movement of the vehicle.

### A.3 Model noise tuning

This noise represents all the approximations made when writing the kinematic model of the compactor front roll. Since the heading estimate error variance has already been fixed through the tuning of $Q_\beta$, we have chosen to set to zero the variance of the model noise added to the heading prediction. After different tests, we obtain the following tuning:

$$Q_\alpha = \begin{bmatrix} (10^{-3})^2 & 0 & 0 \\ 0 & (10^{-3})^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

### References


