Overview

- SLAM large Environments
- CEKF / Sub-Optimal SLAM
- Aiding SLAM with Absolute Information
- Bearing Only SLAM
- Closing large Loops (Hybrid Architecture)
Basic Principle of SLAM

Vehicle and Observation Model

\[
\begin{align*}
\dot{x} &= v_c \cdot \cos(\phi) - \frac{v_c}{L} (a \cdot \sin(\phi) + b \cdot \cos(\phi) \cdot \tan(\alpha)) \\
\dot{y} &= v_c \cdot \sin(\phi) + \frac{v_c}{L} (a \cdot \cos(\phi) - b \cdot \sin(\phi) \cdot \tan(\alpha)) \\
\dot{\phi} &= \frac{v_c}{L} \cdot \tan(\alpha)
\end{align*}
\]

\[
\begin{bmatrix}
\dot{r}_i \\
\dot{\alpha}_i
\end{bmatrix} = h(X_i, x_i, y_i) = \begin{bmatrix}
\sqrt{(x_i - x_i)^2 + (y_i - y_i)^2} \\
\arctan \left( \frac{y_i - y_i}{x_i - x_i} \right) - \phi + \frac{\pi}{2}
\end{bmatrix}
\]

or

\[
\begin{align*}
x_i + r_i \cdot \cos(\alpha_i + \frac{\pi}{2}) - x_i &= 0 \\
y_i + r_i \cdot \sin(\alpha_i + \frac{\pi}{2}) - y_i &= 0
\end{align*}
\]
Extensions to SLAM

\[ X = \begin{bmatrix} X_v \\ X_L \end{bmatrix} \]

\[ X_v = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix}, \quad X_L = \begin{bmatrix} x_i \\ y_i \\ ... \\ x_W \\ y_W \end{bmatrix} \]

- The computational requirements for each update will be proportional to $\mathbf{V}$.

\[ x_v(k+1) = f(x_v(k)) \]
\[ x_v(k+1) = x_v(k) \]

\[ J_1 \in \mathbb{R}^{1 \times 3}, \quad \mathbf{Q} \in \mathbb{R}^{3 \times 3}, \quad I \in \mathbb{R}^{3 \times 3} \]

\[ \frac{\partial F}{\partial X} = \begin{bmatrix} \frac{\partial f}{\partial x_v} & \mathbf{Q} \\ \mathbf{Q}^T & I \end{bmatrix} = \begin{bmatrix} J_1 & \mathbf{Q} \\ \mathbf{Q}^T & I \end{bmatrix} \]

In a large Environment:

Expensive!
Compressed Filter (CEKF)

- **Key Concept:**
  - When the vehicle navigates in a local area observing a group of features, the information gained is a function of only the observed features.
  - This information can be saved and then transferred in one iteration to the rest of the map.
- **Importance of the Compressed Algorithm**
  - Constant Computational Requirements
    - Independent of the total number of features in the global map
  - Full use of High Frequency sensors

CEKF

- The computational cost of the SLAM will now be proportional to $N_a \times N_a$ (landmarks in the local area)
- Full update is only required when the vehicle leaves the local Area $A$. 

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Compressed filter operation

- **External Estimator**
- **Global updates**: low frequency full EKF update.

**Internal estimator** (predictions and observations) at high frequency. Estimator running on a reduced system.

Map Management

- Active landmark
- Passive landmark
- Active sectors
- Hysteresis region
Relative Landmark Representation

Normalized covariance matrix image. Absolute Representation

[Diagram of covariance matrix image]

Normalized covariance matrix image. Relative Representation

[Diagram of covariance matrix image]

Sub-optimal Solutions: De-correlation Algorithms

- In the general case it is possible to de-correlate the covariance submatrices corresponding to two groups of states, Xa and Xb.

\[
P_1 = \begin{bmatrix} A & D & E \\ D^T & B & F \\ E^T & F^T & C \end{bmatrix} \quad A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{m \times n}
\]

\[
P_2 = \begin{bmatrix} A + \alpha \bar{E} & \bar{B} & \bar{F} \\ \bar{E} & \bar{F} \end{bmatrix} \quad P_2 \leq P_1
\]

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De-correlation Procedure

$P = \begin{bmatrix} \alpha & C \\ C^T & \beta \end{bmatrix} = \begin{bmatrix} a+\hat{a} & 0 \\ 0 & \beta+\hat{\beta} \end{bmatrix}$

$\hat{a}, \hat{\beta} \sim \tau = \begin{bmatrix} \hat{a} & -C \\ -C^T & \hat{\beta} \end{bmatrix} \geq 0$

$\alpha_i, \alpha_j = \frac{\sum x_{i,j} | k_{i,j} |}{\sum | k_{i,j} |}, i = j

\hat{\beta}_{i,j} = \frac{\sum x_{i,j} | k_{i,j} |}{\sum | k_{i,j} |}, i = j

$\kappa_{i,j,k} = \kappa_{k,j,i} = 1$

$\hat{a}_{i,j} = \sum_{k,j} | k_{i,j} | \quad \hat{\beta}_{i,j} = \sum_{k,j} | k_{i,j} |$

Selection of Passive and Active States

- Active base landmarks
- Active relative landmarks.
- Passive close relative landmarks.
- Passive far relative landmarks.

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Decorrelation Matrices

Reduced Covariance Matrix
Experimental results

Compressed / Simplifications
Difference in position estimation

Difference in orientation estimation
Outdoor Environment

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SLAM Results

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The aim was to apply monocular video as an external sensor into a simultaneous localization & mapping application. This approach could be so that the camera is a stand-alone external sensor or fused with multiple external sensors.

Modelling the Camera

- Modelled as a calibrated pin-hole camera.

\[
\begin{align*}
\alpha_i &= f_v \tan \phi_i - \tan \left( y_i - y_L \right) \left( x_i - x_L \right) + C_v \\
v_i &= \frac{f_v z_i}{\sqrt{(x_i - x_L)^2 + (y_i - y_L)^2} \cos \beta - \tan \left( y_i - y_L \right) \left( x_i - x_L \right)} + C_v
\end{align*}
\]

- Acts as a bearing-only sensor
Data Association within Pairings

- Without *a priori* information about the landmarks and features in question, image techniques and trigonometry are used to verify the two observations are indeed the same landmark.

- The basis for pairing observations are based that:
  - The estimate of the two lies within view of both observations
  - Are separated in both time and distance
  - Their image patches match by 95% (using Correlation Test)

- Verification of a pairing is performed by a third observation that matches the same criterion as above, as well as performing a $\chi^2$ test from the estimate of the pairing.

Data Association with Known Landmarks

- The handy aspects about the EKF, is that it provides enough information to determine where a landmarks should be seen, through the observation model and the use of validation tests.

- The use of the Mahalanobis distance, or $\chi^2$ test, utilizes the estimated innovation covariance, derived from the EKF. This provides us with a region in which a valid observation can be made

\[
\chi^2 \geq v^T S^{-1} v
\]

- In this case, where we are dealing with a single compatibility of landmark-to-observation for 2 dof. observation, a $\chi^2$ value of 7.38 is appropriate for a 95% confidence
Projecting onto the image

- The $\chi^2$ can be rewritten as the boundary equation for the ellipse it represents, and then used as the dimensions of a search window.

$$\chi^2 \geq v^T S^{-1} v$$

$$\downarrow$$

$$\chi^2 \geq \frac{1}{\det S} \left( S_{22} \Delta u^2 - (S_{12} + S_{21}) \Delta u \Delta v + S_{11} \Delta v^2 \right)$$

Test Environment - Outdoors
Results
• The red line indicates the path from the SLAM algorithm.
• The blue line is the recorded differential GPS path.
Incorporating Absolute Information

- In many cases absolute information is available with different levels of accuracy
  - GPS
  - Landmarks univocally detected at known positions

• Strong correction are possible due to large innovation (long periods with relative information)

• Innovation may not be large but strong updates in the covariance may introduce numerical problems

• A new absolute observation is treated as L observations of quality R/L.
Closing Large Loops: Data Association Problems

Hybrid Architecture

Data Association ERROR

CEKF

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Bayesian Estimation in Navigation

- The Localization Problem with Particle Filters

\[
p(x_{k|k} \mid m, Z^k, U^k, x_0) = \int p(z_k \mid m, x_{k|k}) \int p(x_{k|k} \mid x_{k-1|k}, u^k) p(x_{k-1|k} \mid m, Z^{k-1}, U^{k-1}, x_0) dx_{k-1|k}
\]

- Is possible to develop an algorithm that resolve the localization problem with:
  - Range and Bearing Information
  - Bearing Only Information
  - Range Only Information

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Advantages of Particle Filters

- The localisation problem can be solved without accurate initial vehicle position
- Natural solutions to data association
- The computational issues can be addressed
  - It can adapt to actual computational resources by adjusting the number of particles
  - It can be made very efficient with appropriate distribution of particles

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CEKF Aided by the Particle Filter

• The basic idea:
  – At a certain time the SLAM algorithm will have and association failure. This may be the case when closing a large loop.
  – At this point, we have the actual mean and deviation of the vehicle states (given by the SLAM algorithm).
  – With the actual map we build an uncorrelated map.
  – A particle filter used this information to resolve the position of the rover.

Aiding CEKF

• Implementation Issues
  – Construct an uncorrelated map
  – Algorithm efficiency
    • Obtain a reduced Map
    • Intelligent initialization
Constructing a uncorrelated map

• **Uncorrelated map obtained from a SLAM:**
  – The map is represented in a local frame centered in the area of interest.
  – For this, two beacons can form a base and the rest are referenced to this base.
  – For example numerically...

\[
\begin{bmatrix}
1 & 0.99 & 0.99 \\
0.99 & 1 & 0.99 \\
0.99 & 0.99 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
0.0067 & -0.0033 & -0.0033 \\
-0.0033 & 0.0067 & -0.0033 \\
-0.0033 & -0.0033 & 0.0067
\end{bmatrix}
\]

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Improving the Algorithm Efficiency

This is done in two steps:

– First: select only the “visible beacons” of the entire map taking into account the uncertainties present.

– Fine tuning may discard some landmarks that are known with large uncertainty.

Initialization

– A uniform distribution covering all the area will generate a large number of particles in places with very low probability.

– If we have a set of observations from a laser frame and all the possible beacons that the vehicle can “see”, the sensor can only be over a helical center at each beacon at a distance given by the observation \( r, \theta \), parameterized in \( \tau \).

– This initialization selectively place the particles close to the possible hypothesis.
Location of the particles

\[ r = \sqrt{(x-x_0)^2 + (y-y_0)^2} \]
\[ \phi = \arctan \left( \frac{y-y_0}{x-x_0} \right) + \frac{\pi}{2} \]

\[ C = \bigcup_{i=1}^{N} C_i \]

\[
\begin{align*}
C = & \left\{ (x, y, \phi) \right\} \\
& | \begin{array}{l}
x = x_1 \Rightarrow x + r \cos(\phi) \\
y = y_1 \Rightarrow y + r \sin(\phi) \\
\phi = \phi \Rightarrow \phi - \frac{\pi}{2} \\
r \in [0, 2\pi]
\end{array} \right\}
\]

Initialization with Range and Bearing

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Resampling stage

Particles after resampling

Experimental Results

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