“Stationary” Video

- Last time --- we considered how to model the consistent appearance of a scene, by capturing the local/stochastic distribution at each pixel.
  - Intensity
  - Spatio-temporal derivatives.

- Last time, we made an effort to capture the global regularities in some video sequences.
  - This was based on PCA – linear combinations of images (an average image, plus a weighted combination of “overlays” called principle components).

Sometimes, scene variations are not well modeled by principle components. That is, the scene variation is SIMPLER than the principle component analysis would suggest.

More analytic results – application to MRI images.

Cardiac MRI, courtesy of Nikos Tsekos, Department of Radiology, Washington University Medical School.

Better Distance Function Gives Better Results.

Automatic Registration of complex deformation.

Locally linear embedding (LLE)

- Local neighborhood relationship preserving.
- Uses more than distance between neighbors.

First solve for \( W \) from \( X \).
Then solve for \( Y \) from \( W \).

\[
\hat{X}_i = \sum W_{ij} \hat{X}_j
\]

\[
X = \text{pixels x images}
W = \text{images x images}
\]

\[
Y = \sum W_{ij} Y_j
\]

\[
Y = D \times \text{images}
W = \text{images x images}
\]

\[
W \text{ is very sparse. Seems to be a hard eigenvalue problem to solve.}
\]

Idea, find \( Y \) that minimizes…

\[
\Phi(Y) = \sum_j |Y_j - \sum W_{ij} Y_j|^2
\]
Another Technique: LLE

Comparison to Isomap

Image distance measures that we've seen already.
- Correlation
- Sum of squared pixel intensity difference.

Histogram Example

Histogramming Image Features
- Color
- Texture
- Shape
- Others...
- Create histogram through binning or some procedure to get a distribution
Non-parametric Test Statistics

- Kolmogorov-Smirnov distance (K-S)

\[ D'(I,J) = \max \left| F'(i;I) - F'(i;J) \right| \]

- Cramer-von Mises type (CvM)

\[ D'(I,J) = \sum_i (F'(i;I) - F'(i;J))^2 \]

Cumulative Difference Example

Heuristic Histogram distances

- Or, if you used to be a mathematician:

\[ D(I,J) = \left( \sum_i |f(i;I) - f(i;J)|^p \right)^{1/p} \]

Non-parametric test statistics

The \( \chi^2 \)-statistic is given by

\[ D(I,J) = \sum_i \left( \frac{f(i;I) - \bar{f}(i)}{\bar{f}(i)} \right)^2 \]

(Image similarity with L1 distance)

(Image similarity w. chi-sqr statistics)
Information-theoretic divergences

(i) The Kullback–Leibler divergence (KL) suggested in [10] as an image dissimilarity measure is defined by

\[ D(I, J) = \sum_i f(i; I) \log \frac{f(i; I)}{f(i; J)} . \]  

(ii) The Jeffrey–divergence (JD) is defined by

\[ D(I, J) = \sum_i f(i; I) \log \frac{f(i; I)}{f(i; J)} + f(i; J) \log \frac{f(i; J)}{f(i)} . \]

Problems with Binning

Perceptual similarity

- Quadratic form

\[ D(I, J) = \sqrt{(\bar{f}_I - \bar{f}_J)^T A (\bar{f}_I - \bar{f}_J)} , \]

- Earth Moving Distance

Problem with quadratic norm

Image similarity w. quadratic-form
Image similarity with Earth Moving Distance (EMD)

Earth Moving Distance
- Let P, Q to be 2 histogram signature:
  - P={(P1,w_p1),…,(Pm,w_pm)}
  - Q={(Q1:w_q1),…,(Qn,w_qn)}
- Find a optimal mapping from P to Q
- Define a flow F(i,j) so to minimize

\[
\text{WORK}(P, Q, F) = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} f_{ij}.
\]

Combining different distance measures.
- Affine invariant distance measure:
  - Difference between image P and image Q
  - Find affine matrix A such that
  - \( D(AP, Q) \) is minimized.
  - \( D_{\text{affine}} = |A| \)
  - \( D_{\text{affine-invariant}} = D(AP, Q) \)